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Redistribution and Subsidies for Higher Education*

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Abstract

The financing of higher education through public spending imposes a transfer of resources from taxpayers to university students and their parents. We provide an explanation for this phenomenon. Those who attend institutions of higher education will earn more income in the future and will pay more taxes. People whose children do not receive higher education, however, should agree to help pay the cost of such education, providing that taxes are sufficiently high to ensure an adequate redistribution in favor of their own children at some time in the future.

Keywords: Higher education; taxation; redistribution

JEL classification: *D*71; *H*21; *H*52

I. Introduction

In most countries the cost of public higher education is financed mainly by the government out of general tax revenue. In Spain, for example, in 1996 only about 20 percent of the total cost was covered by students' fees, while the remaining 80 percent was covered by state transfers; see Calero (1996). It is also well documented that most university students come from middle- and upper-income groups. Such empirical evidence has led some authors to conclude that public financing of higher education has a regressive effect on income distribution; see Hansen and Weisbrod (1969) and Nerlove (1972). Leslie and Brinkman (1988), however, conclude that this regressive effect is

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offset once the progressive nature of the tax system is taken into account. Meanwhile, the following question remains to be answered. Since higher education is an activity that provides direct benefits by increasing future earning power, subsidies to higher education imply subsidizing an activity that provides direct benefits only to a privileged minority. Why then, in a democratic society, should a majority of voters, who do not have access to higher education, agree to subsidize it for the wealthiest segment of the population? As this seems to be the case throughout Western societies, many researchers have developed theories to explain this phenomenon.[1]

The explanation most frequently offered is that higher education creates "spillovers," or positive effects, for the rest of the economy. It would, therefore, be unfair to impose the entire cost of higher education on the students and their families. If this were done, families would tend to underinvest in higher education. Since those who receive higher education and generate the spillover cannot force its indirect beneficiaries to pay for the benefits they receive, it can be accomplished through the government, via taxes.

An example in this vein is Johnson (1984), who suggests that education not only increases the productive skills of those who receive it, but can also indirectly benefit those who do not, via "complementarities" in the production process. Creedy and Francois (1990) develop a more complex model. They assume the existence of a positive effect of education on the growth rate of a country. This, in turn, increases the future income of the non-educated individuals who, providing that the positive effect is large enough, will agree to pay higher taxes to continue financing higher education. As Creedy and Francois admit, however, there is no empirical evidence of the existence of such spillovers, and far less of their degree. As a result, this explanation may lose some of its appeal.[2]

Another explanation, offered by Fernandez and Rogerson (1995), is that when people vote on the size of a subsidy for education, they are also voting, implicitly, on how many students should receive such a subsidy and attend university. A given amount of a subsidy determines the proportion of people who can go to university: those above a certain income threshold. The greater the subsidy is, the lower the threshold will be. These authors show that, in some cases, a majority of voters in the middle- and high-income brackets can force the choice of a partial subsidy, and thus exclude low-

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[1] We refer exclusively to "democratic" explanations, i.e., those that are based on some type of collective decision, by majority vote. An alternative explanation for non-democratic societies could be that political power is concentrated in the upper-income group and it can extract the desired resources from the other groups through coercive taxation.

[2] The available empirical evidence suggests that such spillovers are high for primary and secondary education and low for higher education; see e.g. Psacharopoulos (1985).

income groups from higher education while extracting resources from them, through the tax system.

Garratt and Marshall (1994) regard the government as a provider of insurance. Only the most skilled individuals can attend college. If parents are uncertain about the ability of their children, they would be willing to insure themselves against the possibility of having able children while lacking the means to afford their education. Thus, families whose children do not attend college make payments as an insurance premium.

In this paper, we propose a different explanation for public funding of higher education. As mentioned, those who attend university will eventually earn, on average, more income than they would have earned if they had not gone to university. This, in turn, implies that gross total income will increase in the future due to the existence of higher education, and so will the tax base. Moreover, the greater the subsidy to higher education is, the greater the increase in future income will be, as there will be more young people attending university. Let us now suppose that transfers to low-income households are positively related to the tax base. This means that today, families whose children have no access to higher education can anticipate that their children will benefit from it in the future, since the transfers that are beneficial to them will be larger as tax collection increases along with the rise in the number of students. Of course, the higher the marginal tax rate is, the greater the effect will be, as it will determine how much of the increase in income remains in private hands, i.e., in the hands of those who went to university, and how much will be redistributed among the population.[3]

As parents care about the future income of their children, they will be willing to pay the extra taxes needed to finance a higher subsidy. As the level of the subsidy increases, more people will choose to go to university and the tax revenue collected from former university students will also increase.

Our main argument here is that there is a positive relationship between the perceived degree of redistribution of taxes among a population, measured by the marginal tax rate, and the level of the subsidy that is allocated to higher education.[4] The marginal tax rate determines what proportion of the increase in future income due to education is redistributed and how much remains in private hands. This idea is not entirely new. It dates back, at least, to Nerlove (1972) who indicated that part of the subsidy could be regained through

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[3] To avoid problems regarding the formation of expectations of future tax rates, we assume throughout the paper that the entire population believes that the future tax rate will be the same as the current rate.

[4] Following the suggestion of a referee, we verified this positive relationship using cross-country data from the OECD. We found a correlation coefficient of 0.62 between taxes (as measured by the ratio between total tax revenue and GDP in 1998) and subsidies (as measured by the percentage of final funds coming from public sources for tertiary education in 1997). Details are contained in the Appendix.

higher taxes that should be levied on the upper-income groups, i.e., precisely those who have benefited from increased subsidies to higher education. In an optimal taxation framework, Blomquist (1982) studies the extent to which educational expenses should be deductible in order to maximize a Rawlsian social welfare function. Allen (1982) shows that, in some cases, those who are worst off are aided by a linear tax consisting of a wage subsidy and a uniform lump-sum tax, which redistributes from poor to rich. Barr (1993) finds that if the subsidy to higher education were zero, future tax-payers would get a dividend, via the increase in the tax base, and thus the government would eventually have to establish a subsidy to restore efficiency.[5] Our argument is that this could be guaranteed, even in a situation where the subsidy was chosen by majority vote, with the voters trying to maximize their own income levels. To strengthen our point, we consider the decisive voters to be those families who do not, or cannot, send their children to university. Our main finding is that such families would vote for a positive subsidy to higher education, and that such a subsidy would grow along with the level of the marginal tax rate. This contrasts with the results of Fernandez and Rogerson (1995); in their case, the decisive voters were always those who had attended higher education.

In Section II, we introduce the model and some preliminary results. Our model is of the human capital variety, as university education increases productivity. We assume that society has to decide, by majority vote, on the amount of the subsidy to be allocated to higher education. When a family votes, it considers the way in which a given level of subsidy determines the proportion of students that attend university. In Section III, we study the relationship between the level of subsidy chosen by society and the marginal tax rate. We find that, for reasonable values of the parameters, the relationship is always positive. That is to say, the higher the marginal tax rate, the higher the level of the subsidy chosen by society. Finally, in Section IV, we discuss some of the weaknesses of our model.

II. Preliminaries

We set up a model with two periods, labelled 0 and 1. In period 0 there is a continuum of families composed of one parent and one offspring. Families are characterized by a pair (y_0, δ) where $y_0 \in [0, \infty)$ is income earned by the parent in period 0 and $\delta \in [0, \infty)$ is the ability or "talent" of the child. We assume that y_0 and δ are independently distributed. Suppose, for

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[5] A similar idea is explored in Bergstrom and Blomquist (1996), who propose a situation where people could vote for higher subsidies for day care, as it would induce mothers to join the work force and, hence, pay income tax.

instance, that the particular value of δ depends on individual characteristics, such as intelligence, which we assume, *a priori*, to be unrelated to income. We denote by $G(y_0)$ and $H(\delta)$ the cumulative density functions (CDF) of y_0 and δ, respectively, with $g(y_0)$ and $h(\delta)$ as the density functions.

The total cost per student in higher education is $k + c$, where $k > 0$ and $c > 0$. The term k represents the implicit cost, i.e., foregone earnings. For simplicity, we assume that k is independent of the characteristics of the families. The term c represents all explicit costs: tuition, fees, room and board, etc. There is a crucial difference between k and c in our model. The implicit cost k is not subsidized at all, while c can be subsidized by the government at a rate s, with $0 \leqslant s \leqslant 1$. In other words, families pay $k + (1 - s)c$ if their offspring goes to university.

All decisions are made by the parents in period 0. Specifically, they have to make two decisions. The first is whether for a given s, they agree to pay $k + (1 - s)c$. We assume that children are unable to borrow funds and thus cannot afford to pay their own fees, so that they cannot attend university unless their parents are willing to pay for it. Second, parents have to decide collectively, by majority vote, the level of the subsidy that should be allocated to higher education.

In period 1, only the children live and their levels of income are determined by both the former income levels of their parents and their attendance or not at university. We assume that income in period 1, y_1, for a child from a family with characteristics (y_0, δ) will comply with the following pattern:[6]

$$y_1(y_0, \delta) = \begin{cases} y_0 & \text{if the child does not attend university} \\ (1+\delta)y_0 & \text{if the child attends university.} \end{cases} \tag{1}$$

According to this pattern, the value of δ matters only when the child attends university. This amounts to saying that δ is more a measure of ability to learn than a measure of talent. It captures differences in the increment in human capital from attending university.

Tax Structure

Taxes are levied according to the following linear equations:

$$t(y_0) = b_0 + ay_0 \quad \text{where } b_0 \leqslant 0 \text{ and } 0 < a < 1 \text{ in period 0,} \tag{2}$$

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[6] We assume that all students graduate. The return from education does not depend on the number of students. A more general treatment would allow for such a possibility, by e.g. introducing wage adjustments to changes in the supply of graduate students.

and

$$t(y_1) = b_1 + ay_1 \quad \text{where } b_1 \leqslant 0 \text{ and } 0 < a < 1 \text{ in period 1.} \tag{3}$$

The marginal tax rate a is the same during both periods and is fixed by the government. The lump-sum transfers $-b_0$ and $-b_1$ will vary to satisfy the budget constraints of the government (see below). These transfers can be regarded as the guaranteed minimum income for every family. They are assumed to be non-negative to ensure that the tax function is progressive.

Parents' First Decision

Here, we take the subsidy level s as given. We assume that the parents' utility is simply a weighted sum of their after-tax income during period 0 and the after-tax income of their offspring during period 1. If the child attends university, the utility that the parent of a family (y_0, δ) enjoys is:

$$v(s;\, y_0, \delta) = (1-a)y_0 - b_0 - k - (1-s)c + \lambda\{(1-a)(1+\delta)y_0 - b_1\}. \tag{4}$$

From after-tax income during period 0 we subtract $k + (1-s)c$, which represents the share of the cost of higher education that the family has to contribute.

The parameter $\lambda > 0$ expresses the rate at which parents discount their children's income. The greater λ is, the more weight they attach to the future income of their children. This parameter may be regarded as the degree of parental altruism. They will agree to pay λ dollars of their after-tax income during period 0 if, by doing so, the after-tax income of their offspring will increase by 1 dollar during period 1. If $\lambda < 1$, parents attach more weight to their own income than to the future incomes of their children. If the child does not attend university, the utility is:

$$u(s;\, y_0, \delta) = (1-a)y_0 - b_0 + \lambda\{(1-a)y_0 - b_1\}. \tag{5}$$

Parents will agree to pay the cost of university education provided $v(s;\, y_0, \delta) \geqslant u(s;\, y_0, \delta)$. This will be the case when the discounted after-tax increase in their child's income, thanks to higher education, is equal to or greater than the total cost borne by the family. That is:

$$\lambda(1-a)\delta y_0 \geqslant (1-s)c + k. \tag{6}$$

Note that, even when $s = 1$, not all families are willing to send their children to university, as would be the case if $k = 0$. Certainly, if $s = 1$ and $k = 0$, the total cost of education would be zero.

Let $w(s, a) = [(1 - s)c + k]/[\lambda(1 - a)]$. This is a cutoff value for δy_0. The value $(1 - a)w(s, a)$ represents the net cost of education, in future value. Therefore, the families that want their children to attend university are those with the characteristics (y_0, δ) satisfying:

$$\delta y_0 \geqslant w(s, a). \tag{7}$$

As $\lambda > 0$ and $a < 1$, the term $w(s, a)$ is well defined. Note that $w(s, a)$ increases with c, k and a, and decreases with s and λ. Furthermore, for given c, k, λ and a,

$$w(s, a) \in \left[\frac{k}{\lambda(1 - a)}, \frac{c + k}{\lambda(1 - a)}\right].$$

We define $\hat{s}(y_0, \delta)$ as the minimum value of the subsidy s that a family with the characteristics (y_0, δ) requires to be willing to send its child to university. From the above inequality we have:

$$\hat{s}(y_0, \delta) = 1 - \left[\frac{\lambda(1 - a)\delta y_0 - k}{c}\right]. \tag{8}$$

According to the value of $\hat{s}(y_0, \delta)$, the population can be partitioned into three groups:

(i) those with $\hat{s}(y_0, \delta) > 1$, or $\delta y_0 < w(1, a)$;
(ii) those with $\hat{s}(y_0, \delta) < 0$, or $\delta y_0 > w(0, a)$;
(iii) those with $0 \leqslant \hat{s}(y_0, \delta) \leqslant 1$, or $w(1, a) \leqslant \delta y_0 \leqslant w(0, a)$.

Group (i) contains those families that would require a higher subsidy than the maximum ($s = 1$) for their child to have access to higher education. In other words, it contains all the families that will never enjoy higher education. Group (ii) reflects the families that, contrary to (i), will be willing to send their child to university even when $s = 0$. Finally, group (iii) is comprised of families whose decision is not independent of s, as it is for those in groups (i) and (ii). They will only send their child to university if $s \geqslant \hat{s}(y_0, \delta)$.

We can now compute the proportion of families that wish to send their offspring to university. We write this proportion as a function of s and a:

$$p(s, a) = \Pr[z \geqslant w(s, a)] = 1 - F(w(s, a)) = \int_{w(s,a)}^{+\infty} dF(z), \tag{9}$$

where $z = \delta y_0$ and $F(z)$ is the CDF of z, with $f(z)$ as its density function.[7] Note that $p(s, a)$ increases with s and decreases with a. This point encompasses the basic tradeoff of the analysis.

The Government's Budget Constraints

We assume that the government cannot transfer funds from one period to another. The reason is that our two-period model can be regarded as a simplification of the steady state in a multi-period model. We also assume that, apart from redistribution, the only other expenditure is the subsidy to be allocated to higher education. In period 0 the budget constraint is:

$$\int_0^{\infty} (ay_0 + b_0)dG(y_0) = scp(s, a). \tag{10}$$

The term on the right represents total subsidies. From this equation we can obtain the value of b_0 that balances the constraint:

$$b_0(s, a) = scp(s, a) - a\bar{y}_0, \tag{11}$$

where $\bar{y}_0$ is the mean income in period 0. As $b_0(s, a)$ increases in s, to ensure that $b_0(s, a) \leqslant 0$ for all s we must assume that $a \geqslant c/\bar{y}_0$. In other words, we need a lower bound for a that is greater than zero. The fact that $b_0(s, a)$ is increasing in s means that the lump-sum transfer to all families $(-b_0(s, a))$ decreases with s. This implies a reduction in the guaranteed minimum income.

In period 1, the government collects taxes for redistribution purposes only. The value of b_1 that balances the constraint is:

$$b_1(s, a) = -a\bar{y}_1(s, a), \tag{12}$$

where $\bar{y}_1(s, a)$ is the mean income in period 1 and is equal to:

$$\overline{y_1}(s, a) = \overline{y_0} + \int_{w(s,a)}^{+\infty} z\, dF(z). \tag{13}$$

As $\overline{y_1}(s, a)$ increases in s, $b_1(s, a)$ decreases in s. That is, as s increases

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[7] If, for instance, both δ and y_0 follow a log-normal distribution, this is also the case for z. In particular if $\delta \sim LN(\mu_\delta, \sigma_\delta^2)$ and $y_0 \sim LN(\mu_y, \sigma_y^2)$, then $z \sim LN(\mu_z, \sigma_z^2)$ where $\mu_z = \mu_\delta + \mu_y$ and $\sigma_z^2 = \sigma_\delta^2 + \sigma_y^2$.

with a fixed, the guaranteed minimum income in period 1 ($-b_1(s, a)$) will be larger.

Parents' Political Decision

We now ask: what level of subsidy will be chosen collectively? To study this, we assume that when confronted with a choice between two different levels of subsidy, every family will vote for the level that maximizes its indirect utility function. We require the level of subsidy chosen by society to be a Condorcet winner. We call the level s^* a Condorcet winner if, for all $s \neq s^*$, $U(s^*; y_0, \delta) \geqslant U(s; y_0, \delta)$ for at least half of the population.

Let $U(s; y_0, \delta)$ be the indirect utility function of a family with the characteristics (y_0, δ). This function will be different for each of the three types of families characterized according to particular values of $\hat{s}(y_0, \delta)$: first, families with $\hat{s}(y_0, \delta) > 1$ will have $U(s; y_0, \delta) = u(s; y_0, \delta)$ for all $s \in [0, 1]$; second, families with $\hat{s}(y_0, \delta) < 0$, $U(s; y_0, \delta) = v(s; y_0, \delta)$ for all $s \in [0, 1]$; third, for those families with $0 \leqslant \hat{s}(y_0, \delta) \leqslant 1$:

$$U(s; y_0, \delta) = \begin{cases} u(s; y_0, \delta) & \text{for } 0 \leqslant s < \hat{s}(y_0, \delta) \\ v(s; y_0, \delta) & \text{for } \hat{s}(y_0, \delta) \leqslant s \leqslant 1. \end{cases} \tag{14}$$

Empirical evidence suggests that the first group constitutes a majority. This is so when, even in the most favorable case, at least half of the families will not be able to send their children to university. This, in turn, means that the proportion $p(s, a)$ is bounded from above by $\frac{1}{2}$. In Spain, for example, with a subsidy of about 0.8, 77.2 percent of the young people aged 15–24 were not at university in 1995 (OECD average 82 percent). The following assumption introduces some restrictions on the primitives of the model that will enable a result in line with such empirical evidence.

Assumption 1. (i) $f(z)$ is uni-modal. (ii) $Mode(z) \leqslant Median(z) < w(s, a)$.

The fact that $Mode(z) \leqslant Median(z)$, under (i), means that the $Median(z)$ is not in the increasing part of $f(z)$. The condition that $Median(z) < w(s, a)$ guarantees that $p(s, a) < \frac{1}{2}$. To illustrate this assumption, suppose that z is log-normal. Then (i) holds. It is also true that $Mode(z) \leqslant Median(z)$. Moreover, $Median(z) = \exp(\mu_z)$, where μ_z is the mean in logarithms of z. As $w(s, a)$ has values in the interval

$$\left[\frac{k}{\lambda(1-a)}, \frac{c+k}{\lambda(1-a)}\right],$$

the second part of (ii) requires that $a \geqslant 1 - (k/\lambda)\exp(-\mu_z)$. Recall that the progressiveness of the tax system also required $a \geqslant c/\bar{y}_0$. Combining these two restrictions, we have that the marginal tax rate must satisfy

$$a \geqslant a_{\min} = \max\left\{\frac{c}{\bar{y}_0}, 1 - \frac{k}{\lambda}\exp(-\mu_z)\right\}.^{8}$$

If the first group of families makes up at least half of the population, we can prove that a Condorcet winner always exists. To do so, note that all of the members of this group have utility functions as follows:

$$U(s;\, y_0, \delta) = u(s;\, y_0, \delta) = (1 + \lambda)(1 - a)y_0 - b_0(s, a) - \lambda b_1(s, a). \quad (15)$$

Only the first term of the utility function depends on the characteristics of the family. This is due to the additive form of the utility function. All the utility functions of the individuals in this group will therefore reach a maximum at the same value of s, which we call s_1. This will be the Condorcet winner. s_1 is the value that maximizes tax revenue, net of subsidies received, from the group that might send their children to university.

Once we know that a Condorcet winner exists, the next step is to check whether it is strictly positive or not. As the function $u(s;\, y_0, \delta)$ is continuous in the entire interval [0,1], a sufficient condition is that the first derivative of $u(s;\, y_0, \delta)$ at the point $s = 0$ is strictly positive. This will be the case if and only if:

$$a\lambda \frac{\partial \overline{y_1}(0, a)}{\partial s} > cp(0, a). \quad (16)$$

According to this condition, the discounted increase in future tax collection due to a marginal increase in the subsidy must be higher than the increase in the total cost of higher education. If the function $u(s;\, y_0, \delta)$ is strictly concave, this condition is also necessary. For given values of λ and c, the condition will hold whenever the marginal tax rate a lies above some threshold value. In the next section we compute, for reasonable values of the parameters, this threshold value of a, which we call a_l.

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[8] We could allow $k = 0$, but then we would need another restriction to guarantee that a majority of families do not send their children to university. One possibility would be to assume that universities require students to have a minimum level of ability, say $\delta_{\min}$. By assuming that the distribution of ability across the population is such that the proportion of families with a value of δ above $\delta_{\min}$ is less than $\frac{1}{2}$, all of the results in the model can be replicated.

Having a Condorcet winner that is strictly positive is not enough for our intention to perform some exercises in comparative statics. Strict concavity of $u(s;\, y_0,\, \delta)$ would help. The next result gives a condition under which $u(s;\, y_0,\, \delta)$ is strictly concave on s (see Appendix for a proof).

Proposition 1. *Suppose that Assumption 1 holds. Moreover, suppose the following condition holds:*

$$\lim_{z \to +\infty} \left(-\frac{f'(z)}{f(z)} \right) = 0. \tag{17}$$

Then there is some value of the marginal tax rate $\hat{a}$, where $0 < \hat{a} \leqslant 1$, such that if $a \leqslant \hat{a}$, both $v(s;\, y_0,\, \delta)$ and $u(s;\, y_0,\, \delta)$ are strictly concave functions on s.

Consider again the log-normal case. The ratio $-[f'(z)/f(z)]$ is equal to $[\ln(z) - \mu_z + \sigma_z^2]/z\sigma_z^2$, and the condition holds. To illustrate further, if z follows a Pareto distribution with shape parameter $d > 0$, the ratio $-[f'(z)/f(z)]$ is $(d+1)/z$ and the condition also holds. It should be emphasized that $a \leqslant \hat{a}$ is a sufficient, but not necessary, condition for strict concavity.

To illustrate the proposition and the restrictions on the parameters, we use US data from 1989, when median income was \$28,906 while mean income was \$36,250. Accordingly, we specify $y_0 \sim LN(3.36,\, 0.4624)$. With respect to δ we propose a median value of 0.4 and a mean value of 0.5. This implies that college graduates obtain, on average, a wage premium of 50 percent over those who do not graduate. Therefore, $\delta \sim LN(-0.92,\, 0.4463)$ and $z \sim LN(2.44,\, 0.9087)$. The National Center for Educational Statistics collects data on the cost of higher education. In 1989–1990, average undergraduate tuition and fees were \$1,356 in public institutions, while average room and board rates were \$1,513 and \$1,635, respectively.[9] We therefore fix $c = \$4,\, 504$ and set $k = \$8,\, 000$. With these data, $c/\overline{y_0} = 0.124$. We also computed the values of $a_{\min}$ and $\hat{a}$ for different values of λ, as shown in Table 1.

In what follows, we shall assume that the conditions of Assumption 1 and Proposition 1 hold, and that $a \leqslant \hat{a}$.

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[9] See http://nces.ed.gov/pubs/digest97/d97t312.html

Table 1. *Extreme values of a*

λ	$a_{\min}$	$\hat{a}$
0.25	0.124	0.44
0.5	0.124	0.52
0.75	0.124	0.57
1	0.303	0.61

III. Comparative Statics

We begin by assuming that $0 < s_1 < 1$. Therefore s_1 must satisfy the first-order condition:

$$\frac{\partial u(s_1;\, y_0,\, \delta)}{\partial s} = -\frac{\partial b_0(s_1,\, a)}{\partial s} - \lambda \frac{\partial b_1(s_1,\, a)}{\partial s} = 0. \tag{18}$$

According to this condition, s_1 must be such that, at the margin, the negative effect of the subsidy on the guaranteed minimum income of parents counterbalances the positive effect of the subsidy on the average income of children. Another way of writing this condition is:

$$-cp(s_1,\, a) - cs_1 \frac{\partial p(s_1,\, a)}{\partial s} + a\lambda \frac{\partial \overline{y_1}(s_1,\, a)}{\partial s} = 0. \tag{19}$$

In our model, the marginal tax rate can be regarded as a measure of future redistribution. Higher education increases future income, but out of each additional dollar a given individual obtains, she must pay a in taxes. We want to find out whether a higher value of a yields a higher value of the subsidy. To study this effect of a, we apply the implicit function theorem to obtain:

$$\frac{\partial s_1}{\partial a} = -\frac{\dfrac{\partial^2 u(s_1;\, y_0,\, \delta)}{\partial s \partial a}}{\dfrac{\partial^2 u(s_1;\, y_0,\, \delta)}{\partial s^2}}. \tag{20}$$

As we are assuming that $u(s;\, y_0,\, \delta)$ is strictly concave, the sign of $\partial s_1/\partial a$ is the sign of $\partial^2 u(s_1;\, y_0,\, \delta)/\partial s \partial a$. This derivative can be written as:

$$-c\frac{\partial p(s_1,\, a)}{\partial a} - cs_1 \frac{\partial^2 p(s_1,\, a)}{\partial s \partial a} + \lambda \frac{\partial \overline{y_1}(s_1,\, a)}{\partial s} + a\lambda \frac{\partial^2 \overline{y_1}(s_1,\, a)}{\partial s \partial a}, \tag{21}$$

or, in terms of the function $p(s, a)$ only, as:

$$-c\frac{\partial p(s_1, a)}{\partial a} + \lambda\left[w(s_1, a) + a\frac{\partial w(s_1, a)}{\partial a}\right]\frac{\partial p(s_1, a)}{\partial s} + [a\lambda w(s_1, a) - cs_1]\frac{\partial^2 p(s_1, a)}{\partial s \partial a}. \tag{22}$$

The first and second terms are positive. The first term reflects the fact that an increase in a will reduce the proportion of students because families forecast that the post-tax return from higher education will contract. Its effect is positive, since s can be higher without increasing the total budget for higher education. The second term reflects the discounted positive impact on tax collection in the future. The third term is a second-order effect and can be either positive or negative. In general, $[a\lambda w(s_1, a) - cs_1]$ will be negative when a is low, and positive when a is high. The sign of the cross derivative of $p(s_1, a)$ is ambiguous. However, we should expect it to be negative. This means that as a grows, $p(s, a)$ becomes less sensitive to variations in s. In particular, this is the case when z follows a log-normal distribution and $w(s_1, a) > Median(z)$. Then

$$\frac{\partial^2 p(s_1, a)}{\partial s \partial a} = \frac{c}{\lambda(1-a)^2}[f(w(s_1, a)) + f'(w(s_1, a))w(s_1, a)]$$

and the sign of $f(w(s_1, a)) + f'(w(s_1, a))w(s_1, a)$ is the sign of $\mu_z - \ln(w(s_1, a))$.

To sum up, we have two first-order effects that are positive, and one second-order effect whose sign is ambiguous. If the first-order effects dominate, the effect of a on s_1 will be positive. To verify this, we calculated the value of s_1 for different values of the parameters. In every case, we found a positive relation between s_1 and a. In fact, we have not found a single case in which s_1 decreases with a; see Table 2. For each value of λ in the table, we also computed two reference values of a, a_l and a_h. The first, a_l, is the maximum value of a for which s_1 is zero. The second, a_h, is the minimum value of a for which s_1 is one.

Assuming a positive effect of a on s_1, that is, assuming that $s_1(a)$ is a non-decreasing function on a, it is interesting to study the total effect of the marginal tax rate on mean income in period 1, which we can write as $\overline{y_1}(s_1(a), a)$. For the sake of simplicity, and based on the figures in Table 2, we assume that a_l and a_h are such that:

Table 2.[a] *Optimal value of s_1 for different combinations of a and λ*[b]

	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$
$a = 0.124$	0 (0.038)	0 (0.157)	0 (0.288)	
$a = 0.15$	0 (0.036)	0 (0.15)	0 (0.277)	
$a = 0.35$	0.356 (0.0259)	0.07 (0.096)	0 (0.188)	0 (0.284)
$a = 0.45$	0.82 (0.029)	0.622 (0.108)	0.43 (0.188)	0.24 (0.256)
$a = 0.6$	1 (0.0163)	1 (0.085)	1 (0.176)	1 (0.27)
a_l	0.278 (0.024)	0.338 (0.095)	0.381 (0.174)	0.415 (0.2467)
a_h	0.49 (0.03)	0.525 (0.118)	0.551 (0.211)	0.572 (0.295)

[a] The values of the parameters are $c =$ \$4,504, $k =$ \$8,000, $z \sim LN(2.44, 0.95)$. The numbers in parantheses are the proportions of students for the corresponding values of the parameters.
[b] For some entries in the table, the assumption $a \leqslant \hat{a}$ is violated. However, this condition was sufficient, but not necessary, for the utility function to be strictly concave. For those cases where $a > \hat{a}$, we have verified directly (by plotting the corresponding indirect utility function) that the entries in the table are correct.

(i) $a_{\min} \leqslant a_l < a_h \leqslant 1$;
(ii) for all $a \leqslant a_l$, $s_1(a) = 0$;
(iii) for all $a \geqslant a_h$, $s_1(a) = 1$;
(iv) for all a such that $a_l < a < a_h$, $0 < s_1(a) < 1$.

Recall that $\overline{y_1}(s, a)$ was defined in Section II as the integral of some function that does not depend on either s or a. Only $w(s, a)$ depends on such parameters. But then, all combinations (s, a) for which $w(s, a)$ is constant give rise to the same value of $\overline{y_1}(s, a)$.

In the first panel of Figure 1, we have drawn some level curves of $\overline{y_1}(s, a)$ in the space (a, s). They are straight lines with slope

$$\frac{\lambda}{c} w(s, a) = \frac{(1 - s)c + k}{c(1 - a)}.$$

In what follows, we refer to these lines as constant-income lines. Note that the greater a is, the greater the slope of the constant-income lines will be. This means that the increase in s required to offset an increase in a, while leaving $\overline{y_1}(s, a)$ unchanged, increases with a. The bold line in the figure represents $s_1(a)$.

It is interesting to note that, both to the left of a_l and to the right of a_h, the function $\overline{y_1}(s_1(a), a)$ decreases in a. The reason is that, in these intervals, $s_1(a)$ is constant (either at 0 or at 1), and thus, any increase in a lowers mean income in period 1.

Let us define a^+ as the value of a at which $\overline{y_1}(s_1(a), a)$ attains a maximum in the interval $[a_l, a_h]$; see Figure 1. This is always well defined. We want to

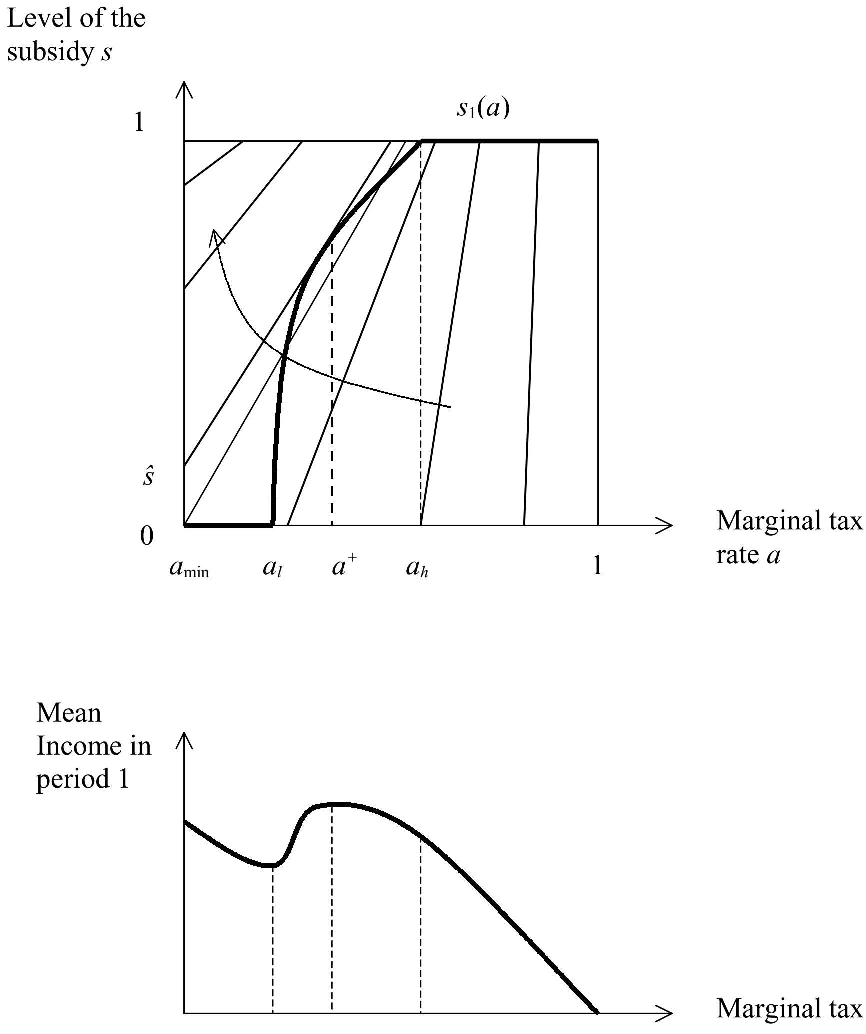

Fig. 1. Relationship between a and mean income in period 1
Note: The arrow in the first panel indicates the direction in which mean income in period 1 increases

check whether a^+ is a global maximum of $\overline{y_1}(s_1(a), a)$ on the interval $[a_{\min}, 1]$. This will not be the case when $a^+ = a_l$. In this case, the maximum of $\overline{y_1}(s_1(a), a)$ will be attained at $a = a_{\min}$ with a subsidy of zero. Let us now suppose that $a^+ > a_l$ as it happens in Figure 1. The following proposition gives a condition under which $\bar{y}_1(s_1(a), a)$ attains a maximum at a^+.

Proposition 2. *Let a_l, a_h and a^+ be as defined above. Then $\overline{y_1}(s_1(a), a)$ will attain a maximum at a^+ if and only if*

$$s_1(a^+) > \left(1 + \frac{k}{c}\right)\left(\frac{a^+ - a_{\min}}{1 - a_{\min}}\right).$$

Proof. All we need to prove is that, under the above condition, $\overline{y_1}(s_1(a), a)$ takes a higher value at a^+ than at $a_{\min}$. Since we already know that $s_1(a_{\min}) = 0$, what we need to prove is that the constant-income line passing through $(a^+, s_1(a^+))$ represents a higher value of $\overline{y_1}(s_1(a), a)$ than the constant-income line passing through $(a_{\min}, 0)$ or, alternatively, that $\hat{s} > 0$ where $\hat{s}$ is defined such that $\overline{y_1}(\hat{s}, a_{\min}) = \overline{y_1}(s_1(a^+), a^+)$. At the point $(s_1(a^+), a^+)$ the slope of the constant-income line will be

$$\beta^+ = \frac{(1 - s_1(a^+))c + k}{c(1 - a^+)}.$$

Therefore, $\hat{s} = s_1(a^+) + \beta^+(a_{\min} - a^+)$. The value $\hat{s}$ will be greater than zero if, and only if,

$$s_1(a^+) > \left(1 + \frac{k}{c}\right)\left(\frac{a^+ - a_{\min}}{1 - a_{\min}}\right).$$

Note, in particular, that this is what happens in Figure 1. ■

For fixed values of $a_{\min}$ and a^+, the condition in the proposition will hold whenever k/c is sufficiently low, in which case the constant-income lines become sufficiently flatter. Recall that k represents the unsubsidized costs. For a high enough ratio between implicit and explicit costs, it may be that the value of a that maximizes $\overline{y_1}(s_1(a), a)$ is $a_{\min}$, at which point we already know that there will be zero subsidy. An increase of the tax rate from $a_{\min}$ to a^+, although it raises s, does not reduce education costs by much. The increase in s reduces only the explicit (subsidized) costs which are, in this case, a small fraction of the total costs. This was the case in the examples of Table 2.

In the second panel of Figure 1 we have shown the values of $\overline{y_1}(s_1(a), a)$ for any value of a, once the voting behavior of the population is taken into account. Note that it is quite similar to a Laffer curve. The difference is that, in our model, the efficiency costs arise, not because of the existence of distortions in the labor supply, but because taxes reduce the demand for education. Moving to the right of a^+ keeps on increasing s, but this increase is not enough to offset the negative impact of the increase in the tax rate.

IV. Final Comments and Criticisms

In our model there are only two ways for parents to transfer resources to their children. One is to pay for the education of their own children. The other is to invest in other families' children and, in that way, influence the size of the guaranteed minimum income the children will obtain in the future. We prove that this second type of investment will be carried out, provided that the future marginal tax rate is high enough. Here, one possible difficulty could be as follows. Suppose parents could transfer resources to their children via cash. Would the results still be maintained? The answer is positive, but this is due to the form of the utility functions. As long as $a > 0$ and $\lambda < 1$, the optimal bequest is always zero. To address this problem properly, we would need a more sophisticated model, at the risk of encountering the problem of non-existence of a voting equilibrium.

The subsidies we have studied are very simplistic. They are conveniently independent of income. In general, however, subsidies are income dependent. Students from low-income families receive grants. Nevertheless, once income has reached a certain level, the subsidy is constant; people in the middle-income group who are not eligible for grants pay the same fees as those in higher, and even much higher income levels. In any case, the introduction of subsidies dependent on income would complicate the problem, since voters would then have to choose policies from a multidimensional space, giving rise to the usual problem of non-existence of equilibrium.

In this paper we have made allowance for people to vote on the amount of the subsidy, while the level of the marginal tax rate remains fixed. If people are also allowed to vote on the level of the marginal tax rate, we would be up against a problem similar to that mentioned above. A possible way out would be sequential voting, that is, voting on one issue at a time. The problem with this is that when we plot the indirect utility functions with respect to the marginal tax rate, we find that these functions are typically not single-peaked, raising a problem of non-existence of equilibrium.

Another related weakness in our model is that voters believe the marginal tax rate will remain unchanged in the future. Note that, in our model, everything depends on the future tax rate rather than on the current rate. What matters most is what the expected future tax rate will be. Adequate treatment of this problem would require a detailed description of how expectations on the future tax rate are formed. In our model, we choose the simplest possible way of doing this by assuming that everyone believes the marginal tax rate will be exactly the same as the current rate.

Interestingly, we find that in our model, the existence of subsidies implies reverse redistribution. To see this, compare two scenarios, one with a zero subsidy and another with a positive subsidy. Then divide families into three groups: (i) those that do not attend college even when the subsidy is positive; (ii) those that attend college only when the subsidy is positive (but not when it is zero); and (iii) those that attend college even with a zero subsidy. To simplify, call these three groups the poor, the middle class and the rich, respectively. What happens in each period when we move from a zero subsidy to a positive subsidy? In the first period, it is obvious that subsidizing education implies reverse redistribution. A minority (the middle class and the rich) get the subsidy while the entire population pays the tax. In the second period, the three groups gain, as the lump-sum transfer rises for all; but the middle class gets an additional benefit since, by attending college, they earn more income. So, overall, it is the middle class that benefits the most in the second period. In net terms, all three groups gain, but the ranking in terms of net gains is: the middle class, then the rich and finally the poor.

In conclusion, the main drawback of the model is its static nature. In a dynamic framework, voters should take into account that the positive effect of s on future income may also, in the next period, increase the proportion of students, thereby making the given subsidy more costly. The rise in the cost of education will reduce the guaranteed minimum income. These two effects have opposite signs and the final result might be ambiguous.

Appendix

Cross-country Evidence on Taxation and Subsidies

Table A1 reports cross-country evidence on taxation and subsidies for several OECD countries. The variable "taxes" refers to the ratio of total tax revenue to GDP in 1998 (1997 for those countries marked with an asterisk). The source is the 1999 edition of *OECD Revenue Statistics*. The variable "subsidy" represents the percent-

Table A1.

	Taxes	Subsidy		Taxes	Subsidy
Australia	30	58	Italy	43	76
Austria	44	86	Japan	29	45
Belgium	46	86	Korea	21	22
Canada*	36	60	Mexico	16	75
Czec Republic	38	86	Netherlands	41	87
Denmark	49	99	Norway	43	93
France	45	85	Portugal	35	98
Germany	37	92	Spain	34	75
Greece*	33	85	Sweden	53	91
Hungary	39	75	United Kingdom	37	73
Iceland	32	94	United States*	30	51
Ireland	32	72			

age of final funds coming from public sources for tertiary education in 1997. The source is the 2000 edition of *Education at a Glance. OECD Indicators*, OECD, Table B2.1, p. 67.

The estimated correlation coefficient is 0.62. We now estimate the following linear regression:

$$S_i = \alpha + \beta T_i + \varepsilon_i, \quad i = 1, \ldots, 23,$$

where S_i is the subsidy, T_i is the tax and ε_i a random disturbance. We obtain $\hat{\alpha} = 26.98$ and $\hat{\beta} = 1.357$, with corresponding standard errors 14.102 and 0.357. The coefficient of the slope is significant at the 0.998 level. Raising the tax burden by 1 percent, implies that the subsidy rises by 1.3 percent.

Proof of Proposition 1

We want to study conditions under which:

$$u''(s) = -c\left(2\frac{\partial p(s, a)}{\partial s} + s\frac{\partial^2 p(s, a)}{\partial s^2}\right) + a\lambda\frac{\partial^2 \overline{y_1}(s, a)}{\partial s^2} < 0. \tag{A1}$$

After substituting the expressions $\partial p(s, a)/\partial s$, $\partial^2 p(s, a)/\partial s^2$ and $\partial^2 \overline{y_1}(s, a)/\partial s^2$ and rearranging terms, this is the same as:

$$\lambda(2 - a)f(w(s, a)) > (sc - a\lambda w(s, a))f'(w(s, a)), \tag{A2}$$

or

$$\lambda(2 - a)f(w(s, a)) > \frac{sc - a(c + k)}{(1 - a)}f'(w(s, a)). \tag{A3}$$

By Assumption 1, $f'(w(s, a)) < 0$. If

$$sc - a(c + k) \geqslant 0 \quad \text{or} \quad a \leqslant \frac{c}{c + k}s$$

we are done. Next, suppose that $sc - a(c + k) < 0$. We have to prove that:

$$-\frac{f'(w(s, a))}{f(w(s, a))} < \frac{\lambda(2 - a)(1 - a)}{a(c + k) - sc}. \tag{A4}$$

Call $B(s, a) = [\lambda(2 - a)(1 - a)]/[a(c + k) - sc]$. We know that for all s and for all $a > [c/(c + k)]s$, $B(s, a) \geqslant 0$, $\partial B(s, a)/\partial a < 0$, $\partial^2 B(s, a)/\partial a^2 > 0$ and $\partial B(s, a)/\partial s > 0$. Moreover $\lim_{a \to [c/(c+k)]s} B(s, a) = +\infty$ and $\lim_{a \to 1} B(s, a) = 0$ for all s. Now let $M(s, a) = -[f'(w(s, a))]/[f(w(s, a))]$. By Assumption 1, $M(s, a) \geqslant 0$ for all (s, a). We also have that $0 < M(s, [c/(c + k)]s) < +\infty$ for all s. By condition (17), we know that $\lim_{a \to 1} M(s, a) = 0$. Now we fix s. As both $B(s, a)$ and $M(s, a)$ are continuous on a there are two possible cases: (i) $B(s, a)$ and

$M(s, a)$ cross at some value (or values) of a; (ii) $M(s, a)$ is below $B(s, a)$ for all a. If they cross, we call $\hat{a}(s)$ the minimum value of a at which those functions cross. In the second case we set $\hat{a}(s) = 1$. Finally we take $\hat{a} = \min_s\{\hat{a}(s)\}$. By construction, it must be that for all $a < \hat{a}$, $u''(s) = v''(s) < 0$. In general, we cannot say anything about the relationship between $\hat{a}(s)$ and s. However, if we consider the log-normal case, then $\hat{a}(s)$ is an increasing function on s and thus, $\hat{a} = \hat{a}(0)$. ■

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