Public Intervention in Education and the Rising College Premium *

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Abstract

College attendance has increased dramatically across the world in the last fifty years. At the same time, the difference between wages of colleges graduates and non-graduates has increased in many countries. One possible explanation of this change in the college premium is that the relative abilities of both groups of individuals have changed. In this paper I build a model trying to explain why these relative abilities may have changed and which could be the effect on the college premium. I find that public intervention has an ambiguous effect in the college premium and, thus, should not be blamed for increasing wage inequality.

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1 Introduction

During the second half of the 20th century many countries experienced a dramatic increase in college attendance. In the United States, for instance, college enrollment increased from 2.1 million in 1951 to 17.1 million in 2004.[1] This increase is the result of several factors such as a decline in tuition costs, improvements in the capital markets that reduced the costs of borrowing, and an increase in the market value of the skills acquired at college. Clearly, many of these factors have been the consequence of a greater involvement of the government in higher education. Declining tuition costs and greater access to borrowing had the effect of relaxing the borrowing constraints for many individuals. While in the past many able individuals did not attend college because they could not afford it, this does not seem to be the relevant case today, at least in the developed countries (Heckman (2000)).

Everything else constant, it seems that the increase in the number of individuals with college education should affect the college premium negatively. In fact, many authors believe that this increase in the supply of college graduates had the effect of depressing the college premium in the 1970s.[2] However, in other periods of time increased enrolment came together with a dramatic rise in the college premium. In the US, between 1979 and 1987 average weekly wages of colleges graduates with 1 to 5 years of experience increased by 30% relative to high school graduates (Katz and Murphy (1992)).[3] The growing college premium raises concerns about the fairness of public intervention in higher education, since an intervention that was originally intended at enhancing equality of opportunities, could end up increasing inequality.

Regarding the increase in the wage gap, there are many competing explanations in the literature. The one with most support seems to be the technology. Technical changes in the last years have favored an increase in the demand of more skilled workers relative to less skilled workers, and in general skilled workers are those with a college education.[4] This explanation has been called the Skill Biased Technical Change (SBTC) hypothesis in the literature. Alternative explanations argue that international trade could have reduced the demand of low skilled workers in some

[1] See NCES, US Department of Education. The website is ⟨http://nces.ed.gov/⟩.

[2] See Murphy and Welch (1992), Katz and Murphy (1992), and Topel (1997).

[3] Also in the United States, according to the Census Bureau, the ratio of mean earnings of college graduates to high-school graduates for full-time male workers rose from 1.41 in 1980 to 1.73 in 1997.

[4] See Acemoglu (2002) and Aghion (2002) and the references therein for a full discussion.

industries,[5] or that the rise in the college premium is mainly due to the fall in the real value of the minimum wage.[6]

Another possible explanation for the increase in the wage gap is due to the existence of composition effects. That is, the increase in the wage gap simply reflects the fact that the average productivity of workers with college education has increased relative to that of workers without college education over time. And this change in relative productivities arises because the type of workers obtaining a college degree is different today than in the past. Then, the average wage for each level of education reflects these composition effects. The change in relative productivities could be justified by the reduction of barriers to entry to college, due to the decline in tuition costs and the reduction in the cost of borrowing. As the constraints that prevent able but poor individuals to attend college disappear, it is expected that the corresponding wages should reflect this fact. As only those individuals with a low skill level are the ones not attending college today, it seems that the college premium must have risen. Since nowadays very few individuals among the young population find difficulties for attending college, firms could infer that, not having a college degree, implies almost surely being low-skilled. This is the line pursued by Hendel, Shapiro and Willen (2005). They conclude by claiming that the removal of barriers to college entry can explain the rise in the college premium. However, the argument is not complete. As many colleagues who are university teachers would agree, increased enrolment in college does not only imply that most of the high-skilled individuals are now attending college, but also that many not so skilled individuals are also benefitting from it. In other words, not only there has been a relaxation of the income cut-off level, but also a significant reduction of the ability cut-off level. If fifty years ago it was true that being rich was a necessary condition for attending college, it was also true that the average skill among the students was high. Thus, I could say that the population of college students could be characterized as being "rich" and "bright". Nowadays we have a situation in which both characteristics have changed. We have a much lower cut-off level for income, but also a much lower cut-off level for ability. And although having a lower income cut-off level goes in the direction of increasing the college premium, since high-ability individuals can attend college today, the fact that

[5] See Borjas and Ramsey (1995).
[6] See Card and DiNardo (2002).

the ability cut-off level is also lower goes in the opposite direction. The distinction is crucial because if it would be true that the changes in the composition of college graduates and non-graduates driven by increased public intervention have the effect of increasing the college premium, we could blame the government for increasing wage inequality. In this paper I build a model that tries to address these issues.

My objective is to test whether composition effects by themselves can explain the rise in the college premium when poor and middle-income individuals are allowed to attend college. The results can be summarized as follows. If ability and income are independent, an improvement of the "quality" of capital markets always generates a rise in the college premium. This result that I obtain is quite general and strengthens the conclusion of Hendel, Shapiro and Willen (2005). However, if the fact that poor and middle-income individuals can attend college is not due to an improvement of capital markets, but because the cost of attending college has reduced due, for example, to a rise in public subsidies, the result is not clear. The college premium can either increase or decrease. Moreover, if ability and income are not independent, the first result can be reversed.

The structure of the paper is the following. In the next section I present the model. In Section 3 I study the effect on the college premium of different changes in the parameters of the model. In Section 4 I explore the situation in which ability and income are not independent. Finally, Section 5 concludes.

2 The Model

There is a mass N of individuals. Each individual is described by two characteristics: innate ability $a \in [\underline{a}, \overline{a}]$ and family income $y \in [0, \overline{y}]$. Ability and income are independently distributed with cumulative distribution functions $G(a)$ and $F(y)$, respectively (density functions are $g(a)$ and $f(y)$). Both $G(a)$ and $F(y)$ are strictly increasing functions. I assume that a is only observable to the individual.

All individuals are compelled by law to attend a minimum level of education. Once the required minimum level is reached, individuals decide whether or not to get post-compulsory education at a college. After attending college, individuals become skilled workers. I call S the number of skilled workers, i.e., the number of college graduates. Those who did not attend college remain unskilled. I call U the number of skilled workers. As there is no unemployment, $S + U = N$.

For simplicity I assume that compulsory education does not increase ability. The productivity at a firm of an individual with only compulsory education is her innate ability a. By attending college, an individual can increase her productivity to βa, where $\beta \geq 1$. The fact that $\beta \geq 1$ implies that high ability individuals are selected into college.

Attending college is costly. First, there is a monetary cost $K \geq 0$. This cost includes fees, tuition, accommodation, and it is net of all possible government subsidies. Second, individuals need to exert some effort at college. The cost of effort is described by $c(a)$, where $c'(a) < 0$, i.e., the higher is ability, the lower is the cost of effort.

In the remainder of this section I study the equilibrium in the labor market. I focus mainly on skilled workers since, as there is no unemployment, as long as the market for skilled workers is in equilibrium, the market for unskilled workers is in equilibrium as well.

2.1 Labor Supply

Individuals choose whether or not to attend college so as to maximize lifetime wealth. Lifetime wealth corresponds to the discounted value of wages, less the cost of education. As workers can be either skilled or unskilled, I refer to the corresponding wages as w_S and w_U, respectively. The college premium is denoted by ω, where:

$$\omega = w_S - w_U. \tag{1}$$

I consider the decision of whether to attend college or not for a given individual with innate ability a and income y. The first thing to note is that the individual must have enough resources to afford the monetary cost K. I assume that individuals can use not only their income y but also, if that income is not enough, they can borrow from a bank. As I want to allow for imperfections in the capital markets in a extremely simple way, I assume that banks will be willing to give a loan of at most D, where $0 \leq D \leq K$. This parameter D captures the "quality" of capital markets. The case $D = 0$ means complete impossibility of borrowing. The case $D = K$ means that capital markets are perfect.[7] Then, a necessary condition for attending college is

[7] The size of the loan D cannot be conditioned on ability a, since banks do not observe ability.

that family income y plus the loan D must be greater than K, or:

$$y \geq K - D. \tag{2}$$

As ability and income are independent, the probability of y being higher than $K-D$ is simply $1-F(K-D)$. To simplify notation further, I will write $p = 1-F(K-D)$. Then, p is the proportion of individuals that can afford a college education. Throughout the paper I will assume that $0 < p \leq 1$, that is, there are always some individuals who can pay K.[8] Note also that p rises with D and decreases with K. When $D = K$, $p = 1$.

Condition (2) above is a necessary condition for attending college. To decide on attendance, individuals compare income if they attend college and income if they do not attend. If an individual decides not to attend college, and assuming for simplicity a zero discount rate, she gets income $w_U + y$. If she attends, she gets $w_S - c(a) - K + y$. Here I am also assuming that the loan pays no interest. The point is that all capital market imperfections are summarized by D. Then, an individual with ability a will attend college provided:

$$\omega - K \geq c(a). \tag{3}$$

For given values of ω and K, Equation (3) defines implicitly a cut-off level for ability that I call α. It is the value that solves (3) with equality. That is,

$$\omega - K = c(\alpha). \tag{4}$$

Only individuals with ability above α attend college.[9]
Since the function $c(\cdot)$ is invertible, I can write α as a function of the college premium as follows:

$$\alpha(\omega) = c^{-1}(\omega - K). \tag{5}$$

I am interesting in deriving an expression for the supply of both skilled and unskilled workers as a function of the college premium ω. As explained above, an individual will become a skilled worker if her income is above $K - D$ and her ability is above α.

[8] This amounts to say that $K < \overline{y}$.

[9] The assumption that the interest rate on loans is zero simplifies a lot the analysis. If there is a strictly positive interest rate R_L, Condition (2) becomes:

$$w_1 - c(a) - K \geq w_0 + R_L(K - y),$$

where now we have that α depends on y. In particular, we could write $\alpha(y)$, with $\alpha'(y) < 0$.

For a given value of the college premium ω, the supply of skilled workers $S(\omega)$ will be, therefore:

$$S(\omega) = \begin{cases} 0 & \text{if } \omega < c(\overline{a}) + K \\ pN\left[1 - G(\alpha(\omega))\right] & \text{if } \omega \geq c(\overline{a}) + K. \end{cases} \tag{6}$$

Note that $0 \leq S(\omega) \leq pN$ and, since there is no unemployment, $(1-p)N \leq U(\omega) \leq N$, where $U(\omega)$ is the supply of unskilled workers. The case $\omega < c(\overline{a}) + K$ corresponds to a situation where, even that individual with the lowest cost of effort $(\overline{a})$ does not go to college. As ω gets higher, $S(\omega)$ also rises since $\alpha(\omega)$ is a decreasing function of ω. It is also interesting to see that an increase in p shifts $S(\omega)$ to the right.

2.2 Labor Demand

Now I want to obtain an expression for the college premium that firms are willing to pay as a function of the number of skilled workers in the economy, i.e., an expression for the indirect demand of skilled workers where the endogenous variable is the college premium ω and the exogenous variable is the number of skilled workers S. This indirect demand function, together with the supply function obtained above (Expression (6)), will define the equilibrium in the labor market.

I assume that there are many firms who employ both skilled and unskilled workers. They observe schooling but not ability. They also know the support of a and y, the cumulative distribution functions $G(a), F(y)$, and the value of β. Firms will offer to an skilled worker a wage that corresponds exactly with the expected productivity among skilled workers which, in turn, will coincide with the expected productivity among college graduates. The wage offered to unskilled workers will be the expected productivity among those with only compulsory education. In particular, the wage paid to college graduates is:

$$\begin{aligned} w_S(\alpha) &= E(\beta a \mid a \geq \alpha \text{ and } y \geq K - D) = \\ &= \beta E(a \mid a \geq \alpha), \end{aligned} \tag{7}$$

where $E(\cdot \mid \cdot)$ denotes the conditional expectation. Since firms observe S and they know p, they infer that the value of α is:

$$\alpha(S) = G^{-1}\left(1 - \frac{S}{pN}\right). \tag{8}$$

Then I can write the skilled wage as a function of S as:

$$w_S(S) = \beta E(a \mid a \geq \alpha(S)), \tag{9}$$

where $\alpha(S)$ follows the above expression. Equation (9) is the inverse demand of skilled workers. As expected, $w_S(S)$ is a decreasing function of S because, the higher is S, the lower is the threshold level α and, thus, the lower is the average productivity among skilled workers. When $S = 0$, w_S takes its maximum value which is $\beta\overline{a}$, and when $S = pN$ it takes its minimum value which is $\beta E(a)$, where $E(a)$ is the unconditional expectation. An increase in p shifts to the right the inverse demand, that is, the higher is p the more are firms willing to pay for any given number of skilled workers S.

For unskilled workers firms must take into account that possibly some able individuals remained unskilled because they could not afford a college education. I will consider first the extreme case in which $p = 1$. If $p = 1$, all individuals can afford college. Then, w_U is just the expected ability of those below the threshold α, that is, $E(a \mid a < \alpha)$. In this case the demand for unskilled labor is upward sloping with respect to the unskilled wage w_U, a fact that might seem counterintuitive. However, the intuition is quite easy. As U gets lower, the cut-off level of ability α gets lower as well, which means that the average productivity among the unskilled is also lower. In particular, when $U = 0$ $(S = N)$ w_U takes the value $\underline{a}$. When $U = N$ $(S = 0)$, w_U takes the value $E(a)$. As $S = N - U$, we can see also that w_U is a strictly decreasing function of S.

The most interesting case is when $0 < p < 1$. In this case I will first derive an expression for w_U as a function of the threshold level α and of p. As we will see below, using Equation (8) it is immediate to derive a similar expression for w_U as a function of either U or S. I have that:

$$\begin{aligned} w_U(\alpha; p) &= E(a \mid a < \alpha \text{ or } a \geq \alpha \text{ and } y < K - D) = \\ &= \frac{G(\alpha)E(a \mid a < \alpha) + (1-p)(1-G(\alpha))E(a \mid a \geq \alpha)}{G(\alpha) + (1-p)(1-G(\alpha))} = \\ &= \frac{pG(\alpha)}{1-p+pG(\alpha)}E(a \mid a < \alpha) + \frac{1-p}{1-p+pG(\alpha)}E(a). \end{aligned} \tag{10}$$

Here w_U can be seen as a weighted average between the expected productivity of those who do not get a college education because their ability is low $(E(a \mid a < \alpha))$ and the expected productivity of those who remain unskilled because they cannot afford college $(E(a))$. Using Equation (8) I can write w_U as a function of U as follows:

$$w_U(U) = \frac{NE(a) - (N-U)E(a \mid a \geq \alpha)}{U}. \tag{11}$$

In the next proposition I list some interesting properties of the function $w_U(U)$.

Proposition 1 *Assume* $0 < p < 1$ *and* $g(a) > 0$ *for all* $a \in [\underline{a}, \overline{a}]$. *The function* $w_U(U)$ *has a U-shape with respect to* U. *In particular, the following properties hold:*
(i) $w_U(U = (1-p)N) = w_U(U = N) = E(a)$.
(ii) $w_U(U) < E(a)$ *for all* U *such that* $(1-p)N < U < N$.
(iii) The function $w_U(U)$ *has a U-shape. It attains a minimum at some* $\widetilde{U}$, *where* $(1-p)N < \widetilde{U} < N$.

Proof. See the Appendix.

This result is quite interesting and contrasts dramatically with the case in which $p = 1$. When $p = 1$ we saw that, as U decreases, w_U reduces accordingly. When $0 < p < 1$, w_U behaves differently. When the initial value of U is large, it is still true that a reduction in U entails a reduction in w_U. However, once U reaches the critical value $\widetilde{U}$, a further reduction in U rises w_U. The reason is that, as p is fixed, the reduction in U must be due to a reduction in α which, in turn, implies that in Equation (10) the weight of $E(a \mid a < \alpha)$ gets lower while the weight of $E(a)$ rises. To the left of $\widetilde{U}$ firms correctly understand that most unskilled workers come from the group that could not pay college. Again, it is clear that an increase in p shifts downward the function $w_U(S)$.

To illustrate the shape of $w_S(S)$, $w_U(U)$ and ω, I present an example:

Example 1 *Assume that* $a \sim U[0,1]$. *Then, I have:*

$$w_S(\alpha) = \beta\left(\frac{1+\alpha}{2}\right). \tag{12}$$

Writing w_S *as a function of* S, *I have:*

$$w_S(S) = \beta\left(1 - \frac{S}{2pN}\right). \tag{13}$$

Assuming $0 < p < 1$:

$$w_U(\alpha; p) = \frac{1 - p(1-\alpha)(1+\alpha)}{2(1 - p(1-\alpha))}. \tag{14}$$

I can also write w_U *as a function of* U *as follows:*

$$w_U(U) = \frac{\frac{N}{2} - (N-U)\left(1 - \frac{1}{2p} + \frac{U}{2pN}\right)}{U}. \tag{15}$$

Alternatively, w_U can be expressed as a function of S as:

$$w_U(S) = \frac{N - S\left(2 - \frac{S}{pN}\right)}{2(N-S)}. \tag{16}$$

I check that $\widetilde{\alpha} = w_U(\widetilde{\alpha}; p) = \frac{-(1-p)+\sqrt{1-p}}{p}$, and that $\widetilde{U} = N\sqrt{1-p}$. For example, take $N = 1$. When $p = 1/2$, $\widetilde{\alpha} = 0.41$ and $\widetilde{U} \approx 0.7$. When $p = 3/4$, $\widetilde{\alpha} = 1/3$ and $\widetilde{U} = 0.5$.

Once I have an expression for both w_S and w_U as a function of S, I can construct an expression for the college premium that firms are willing to pay as a function of the number of skilled workers. This can be seen as a kind of inverse demand function. The college premium that firms are willing to pay when the number of skilled workers is S is:

$$\begin{aligned} \omega(S) &= w_S(S) - w_U(S) = \\ &= \frac{1}{N-S}\left[(\beta(N-S)+S)E\left(a \mid a \geq \alpha(S)\right) - NE(a)\right]. \end{aligned} \tag{17}$$

I represent $\omega(S)$ in Figure 1, together with the skilled labor supply. The function $\omega(S)$ is decreasing with S, reflecting the fact that firms understand that the higher is S the lower is the difference in the expected productivity of the two groups of workers. When $S = 0$, the college premium takes its highest value $\omega(S = 0) = \beta\overline{a} - E(a)$. The lowest value of the college premium is $\omega(S = pN) = (\beta - 1)E(a)$.

2.3 Labor Market Equilibrium

Since there is no unemployment, I can focus only on the market for skilled workers. An equilibrium will be made up by a value of the college premium ω^* and a number of skilled workers S^* such that the supply and the demand of skilled workers cross each other in the space (S, ω), that is:

$$\omega^* = \frac{1}{N-S^*}\left[(\beta(N-S^*)+S^*)E\left(a \mid a \geq \alpha(S^*)\right) - NE(a)\right], \tag{18}$$

and:

$$S^* = pN\left[1 - G(c^{-1}(\omega^* - K))\right]. \tag{19}$$

Once I have the equilibrium values ω^* and S^*, I can calculate the equilibrium values of the skilled wage w_S^* and the unskilled wage w_U^*, respectively. As it is clear in Figure 1, a sufficient condition for having an interior equilibrium is:

$$\beta\overline{a} - E(a) > c(\overline{a}) + K. \tag{20}$$

This condition simply states that the cost of attending college for the individual with the highest ability is low enough. Otherwise I would have an uninteresting equilibrium in which $S^* = 0$.

There is an alternative and interesting way of defining an equilibrium by means of the equilibrium value of the cut-off level of ability α. For a given value of p, and using Equation (4), I can define an equilibrium of this economy as a value of the threshold level α, that I call α^*, such that:

$$\omega(\alpha^*) = w_S(\alpha^*) - w_U(\alpha^*; p) = c(\alpha^*) + K, \tag{21}$$

where $w_S(\cdot)$, and $w_U(\cdot; p)$ are defined as in Subsection 2.2. This represents an alternative characterization of the equilibrium since, once we know α^*, we can use Equations (9) and (10) to obtain the corresponding equilibrium wages, and with Equation (6) I get the equilibrium value of S^*.

As long as $p \neq 1$, in the equilibrium there is no perfect sorting by ability. It is an equilibrium similar to the one in Bedard (2001). Perfect sorting would mean that, when an individual with ability a goes to college and becomes an skilled worker, all individuals with ability $a' > a$ will do so as well. Here, among unskilled workers there are individuals with ability in excess of α^* who are constrained from going to college. However, all individuals attending college must have ability above α^*.

Once I have an equilibrium, I want to perform some exercises on comparative statics. In particular, I want to see how changes in the parameters of the model (β, D, and K) affect the levels of employment of skilled and unskilled workers and the college premium. The kind of comparative exercises in which I am interested are those that have as an unambiguous effect a positive effect on college attendance and, thus, in the number of skilled workers in the economy. As we will see below there are three cases in which this positive effect on college attendance arises: (i) An increase in β; (ii) An increase in D; (iii) A reduction in K. Each of them has a completely different interpretation. Case (i) refers to changes in technology that drive an increase in the demand for skilled workers. Cases (ii) and (iii) refer to changes in the supply of skilled workers. Case (ii) refers to an improvement in the quality of capital markets. Case (iii) could be interpreted as greater government involvement in the financing of college education, through higher subsidies for instance. In the next section I explore these three cases in detail.

3 Comparative Statics

The first exercise of comparative statics is quite straightforward. Using Figure 1 it is clear that an increase in β has a positive effect both on S^* and ω^*. An increase in β has no impact on the supply of skilled workers but it shifts upward its demand. This result is in line with the widespread view among labor economists that technical change favors skilled workers, and increases inequality (Acemoglu (2002)).

The second exercise refers to an improvement in the quality of capital markets. This improvement is described by means of an increase in the parameter D which, in turn, implies that p will rise. Both the supply and the demand shift to the right. The effect on supply comes from the fact that now more individuals can afford college. The effect on demand reflects the fact that now, firms expect a higher average productivity among skilled workers. As a result, S^* will unambiguously increase. The effect on ω^* is not that clear in the picture. Nevertheless, the next proposition proves that an increase in p has always a positive effect on ω^*.

Proposition 2 *Let $0 < p < 1$. Then, an increase in p has always a positive effect in the equilibrium value of the college premium ω^*.*

Proof. I use the definition of α^* from Equation (21) to prove the result. Looking at Equation (9) it is clear that an increase in p does not affect the term $w_S(\alpha)$. On the contrary, I see that $w_U(\alpha; p)$ shifts downward. To see this, I simply calculate the derivative of $w_U(\alpha; p)$ with respect to p :

$$\frac{\partial w_U(\alpha; p)}{\partial p} = \frac{1 - G(\alpha)}{(1 - p(1 - G(\alpha)))^2} \left[E(a) - E(a \mid a \geq \alpha) \right] \leq 0. \tag{22}$$

This, in turn, implies that the whole function $w_U(\alpha; p) + c(\alpha) + K$ shifts downward. As a result of this, using Equation (21) and the fact that $w_S(\alpha)$ rises with α the equilibrium value α^* gets lower. I call α^{**} the new equilibrium, where $\alpha^{**} < \alpha^*$. Now I see what happens with the college premium at the new equilibrium. In the initial equilibrium I had $\omega(\alpha^*) = c(\alpha^*) + K$ while after the change I have $\omega(\alpha^{**}) = c(\alpha^{**}) + K$. As $c(a)$ is a strictly decreasing function, $c(\alpha^{**}) > c(\alpha^*)$ and, thus, $\omega(\alpha^{**}) > \omega(\alpha^*)$. ■

According to this result, better capital markets have a positive impact on the college premium. This result is in line with previous results by Hendel, Shapiro, and

Willen (2005). However, as I shall prove in Section 4, this result relies heavily on the assumption that ability and income are independent of each other.

An interesting lesson from Proposition 2 is that an improvement in the quality of capital markets has always the effect of reducing the cut-off level of ability. When the income constraint relaxes, the ability constraint relaxes as well. As a result of this, the equilibrium wage of college graduates is lower than before. The average productivity among college graduates is lower because less able individuals that were excluded from college before the change in p, now can attend college. Firms respond by adjusting the wages that they are willing to pay. Note that the new entrants are imposing a negative externality on those individuals that attended college before the change.

With respect to the equilibrium wage of those without a college education, I see that it gets lower as well. This comes immediately from the fact that the college premium rises. As p rises, there are two effects. First, the average skill among those who lack a college degree gets lower. The reason is that there are some highly skilled individuals that now can attend college and thus are leaving the pool of individuals without a college education. Second, the pool of individuals without a college degree becomes smaller. As the equilibrium wage for unskilled workers gets lower, what happens is that the change in quality offsets the change in quantity.

The third exercise explores the effect of a reduction in college fees, K. I consider this to be the most interesting of the three exercises, since the decline in the tuition costs paid by the students and their families has been always considered as the main reason for the increase in college attendance. The reduction in tuition costs has been mainly due to higher public involvement in college education. This is what happened in many countries during the 20th century. In fact, in some European countries college is so heavily subsidized that tuition cost is zero. At first sight, it might seem that a direct reduction of K would have the same effect that an improvement in the quality of capital markets, since it has a positive impact on p. However, it turns out that this need not be the case. The reason is the following. A reduction in K has two separate effects. It has an indirect effect through p, which is similar to the effect of an increase in D. As K gets lower, p rises reflecting the fact that now a higher proportion of individuals can afford a college education. Again, both the supply and the demand shift to the right. If this were the only effect I would have exactly the same result as in the case of an increase in D, namely, that the college premium rises. But now there is another direct effect that affects only the supply of skilled labor. This effect comes from the fact that even if p would not change, the monetary value of the alternative of going to college is lower, implying that the cut-off level of ability α is pushed even farther to the left. It is clear that the impact on the supply is stronger than the impact on demand which, in turn, could imply that the college premium need not increase. This could be seen also by looking at Equation (21). Assume that the initial equilibrium is $\omega(\alpha^*) = c(\alpha^*) + K$. Now fees are reduced to $K' < K$. After this change the new equilibrium becomes $\omega(\alpha^{**}) = c(\alpha^{**}) + K'$, where I know that $c(\alpha^{**}) > c(\alpha^*)$. It is clear that, in general, we cannot say anything about the sign of $\omega(\alpha^*) - \omega(\alpha^{**})$. To give an intuition of this result, I use Equation (21) to derive the effect of a change in K :

$$\frac{\partial \omega(\alpha^*)}{\partial K} = c'(\alpha^*)\frac{\partial \alpha^*}{\partial K} + 1. \tag{23}$$

Since $c'(\alpha^*) < 0$ and $\frac{\partial \alpha^*}{\partial K} > 0$ we cannot sign $\frac{\partial \omega(\alpha^*)}{\partial K}$. If we assume $c(a)$ to be a convex function, we could expect that for large values of K, its sign will be negative because in such a case α^* will be large and $c'(\alpha^*)$ will be small. However, once K becomes sufficiently small $c'(\alpha^*)$ will become large, and the sign of $\frac{\partial \omega(\alpha^*)}{\partial K}$ could turn positive. Next I present an example to illustrate this result.

Example 2 *Suppose that income y follows a Lognormal distribution with parameters* $(-0.22, 0.668)$. *I am taking this particular values since I am normalizing mean income to 1, and I assume median income is 0.8. Ability follows a Uniform distribution on the interval* $[0, 1]$. *I assume* $\beta = 1, D = 0, N = 1$. *The cost function is* $c(a) = \frac{\delta(1-a)}{a}$, *where I take* $\delta = 0.1$. *I consider several different values of K and I compute the corresponding equilibrium values of ω, S, α. I also compute the value of $c'(\alpha^*)$. Note that Condition (20) requires $K < 0.5$. The numerical results are gathered in the following table:*

K	p	ω^*	S^*	α^*	$c'(\alpha^*)$
.45	*.81*	*.476*	*.17*	*.79*	*-.16*
.4	*.85*	*.459*	*.32*	*.63*	*-.25*
.35	*.89*	*.451*	*.45*	*.50*	*-.40*
.3	*.93*	*.451*	*.56*	*.40*	*-.63*
.25	*.96*	*.46*	*.65*	*.32*	*-.96*

This example suggests that, in general, it is not true that a reduction in tuition costs has the effect of increasing the college premium. In particular, in the example I observe that college premium follows a U-shape with respect to the tuition cost.

4 Ability and Income are not Independent

In this section I drop the assumption that ability and income are independent, and I explore whether the result of Proposition 2 still remains true. I use a simple example in which both income and ability take only discrete values. In particular, I assume that there are three groups of individuals: The poor, the middle-class, and the rich. The corresponding levels of income are y_L, y_M, and y_H, where $y_L < y_M < y_H$. All poor individuals have ability a_L, all middle-income individuals have ability a_M, and all rich individuals have ability a_H, where $a_L < a_M < a_H$. The three groups represent proportions p_L, p_M, and p_H, respectively. I assume that $p_H \leq p_L < 1/2$, and that $p_L + p_M > 1/2$. These two assumptions guarantee that a_M is the median of the ability distribution. Finally, I assume that the ability distribution is right-skewed, that is $a_M < \overline{a}$, where $\overline{a} = p_L a_L + p_M a_M + p_H a_H$ is the mean ability. Individuals with different abilities need to exert different effort levels in order to attend college. I assume $c(a_L) = +\infty, c(a_M) = c > 0$, and $c(a_H) = 0$. The poor are, therefore, always excluded from college.

In the initial situation the monetary cost of attending college is $K \geq 0$ while the quality of capital markets takes a value D such that only the rich (and highly skilled) can afford college. This amounts to assume that:

$$y_M < K - D \leq y_H. \tag{24}$$

Only rich and bright individuals attend college. To have an equilibrium with these characteristics, I need the following conditions. First, note that in this case $w_S^* = \beta a_H$. Unskilled individuals can be either of ability a_M or a_L. The wage paid to them is:

$$w_U^* = \frac{p_M a_M + p_L a_L}{p_M + p_L}. \tag{25}$$

Second, for this to be an equilibrium, I need the additional condition that individuals with ability a_H are indeed willing to attend college. That is, Condition (3) must hold for them. This requires:

$$\beta a_H - K \geq \frac{p_M a_M + p_L a_L}{p_M + p_L}, \tag{26}$$

since $c(a_H) = 0$. If this situation is an equilibrium, the college premium will be:

$$\omega^* = \beta a_H - \left(\frac{p_M a_M + p_L a_L}{p_M + p_L} \right). \tag{27}$$

Now suppose that there is an improvement in capital markets, and the parameter that represents quality rises to a value D' such that now also middle-income individuals (who have a medium level of ability) can attend college. This means that I require:

$$y_M \geq K - D'. \tag{28}$$

In the sequel I will assume that Conditions (24) and (28) hold.[10] To have an equilibrium in which individuals with ability levels a_H and a_M attend college, again two conditions must be satisfied. First, note that wages are:

$$w'_S = \beta \left(\frac{p_H a_H + p_M a_M}{p_H + p_M} \right), \tag{29}$$

and $w'_U = a_L$, since now only the poor are excluded from college, and all of them have ability a_L. Second, this will be an equilibrium if Condition (3) holds for individuals with ability levels a_H and a_M, respectively. This requires that:

$$\beta \left(\frac{p_H a_H + p_M a_M}{p_H + p_M} \right) - K \geq a_L, \tag{30}$$

and:

$$\beta \left(\frac{p_H a_H + p_M a_M}{p_H + p_M} \right) - c - K \geq a_L. \tag{31}$$

It is clear that Condition (31) implies Condition (30). In turn, if this is an equilibrium, the college premium will be:

$$\omega' = \beta \left(\frac{p_H a_H + p_M a_M}{p_H + p_M} \right) - a_L. \tag{32}$$

To sum up, I need values of the parameters of the model such that Conditions (26) and (31) hold. Consider the following assumption on the parameters.

Assumption 1: $\beta a_H - a_L \geq (c + K) \left(\frac{p_H + p_M}{p_H} \right)$.

One interpretation of this assumption is that the difference between the high and the low levels of skill is large enough. Then, I have the next result (see the Appendix for a proof):

[10] Conditions (24) and (28) will be satisfied as long as:

$$K - D' \leq y_M < K - D \leq y_H.$$

Proposition 3 *Under Assumption 1, for any value of a_M such that $a_L < a_M < a_H$, both Conditions (26) and (31) hold.*

Finally I want to study the change in the college premium induced by the improvement of capital markets. This amounts to compare ω^* and ω'. In the next proposition I describe the conditions under which the college premium gets lower once capital markets improve their quality, meaning that the degree of inequality among skilled and unskilled workers diminishes.

Proposition 4 *Suppose Assumption 1 holds. Suppose also that $p_L \geq p_H$ and that the distribution of ability is right-skewed, i.e. $a_M < \overline{a}$. Then, the college premium after the improvement in capital markets is always lower than before the change.*

Proof. Note that $\omega' < \omega^*$ if and only if:

$$a_M < \frac{\beta(p_L + p_M)a_H + (p_M + p_H)a_L}{p_M + p_H + \beta(p_L + p_M)}. \tag{33}$$

Since the ability distribution is right-skewed, it is true that:

$$a_M < \frac{p_L a_L + p_H a_H}{p_L + p_H}. \tag{34}$$

I have to prove that:

$$\frac{p_L a_L + p_H a_H}{p_L + p_H} < \frac{\beta(p_L + p_M)a_H + (p_M + p_H)a_L}{p_M + p_H + \beta(p_L + p_M)}. \tag{35}$$

Rearranging, this is the same than:

$$\begin{aligned} & a_H \left[\beta\left(1 - p_M\right)\left(p_L + p_M\right) - p_H(1 - p_L + \beta(p_L + p_M))\right] \\ > \; & a_L \left[p_L(1 - p_L + \beta(p_L + p_M)) - \left(1 - p_M\right)\left(1 - p_L\right)\right]. \end{aligned} \tag{36}$$

But clearly, both terms in brackets are exactly the same. So as long as both terms are positive, the result follows because $a_H > a_L$. The terms in brackets are positive if and only if β is above a given threshold:

$$\beta > \frac{p_H(1 - p_L)}{p_L(p_L + p_M)}. \tag{37}$$

But as $p_L \geq p_H$:

$$\frac{p_H(1 - p_L)}{p_L(p_L + p_M)} \leq 1, \tag{38}$$

which guarantees that β is above the threshold. ■

The assumption that the distribution of ability is right-skewed is quite standard. Assuming $p_L \geq p_H$ does not seem quite strong either. This example proves that, when ability and income are not independent, an increase in college attendance due to the arrival of students from middle-income families could easily reduce the college premium.

5 Conclusions

In this paper I build a model in which the equilibrium value of the college premium depends on several supply and demand factors. Since I am particularly interested in how changes in the supply of college workers affect the college premium I concentrate in two exercises of comparative statics. First, if there is a reduction in the cost of borrowing we see that, provided family income and ability are independent variables, the effect on the college premium is always positive. If, however, as it is usually the case family income and ability are not independent variables, the effect could be reversed. Second, a reduction of tuition costs could have either a positive or a negative effect on the college premium. As a general conclusion, it seems that there is weak evidence supporting the fact that the intervention of the government could have the effect of raising wage inequality between college graduates and individuals without a college education.

Appendix:

Proof of Proposition 1

(i) When $U = (1-p)N$ we have $S = pN$ and, by Equation (8), $\alpha = \underline{a}$ implying $E(a \mid a \geq \alpha) = E(a)$. By Equation (11), $w_U = E(a)$. When $U = N$, from Equation (11) it follows immediately that $w_U = E(a)$.

(ii) As $U > (1-p)N$, $S < pN$, then $\alpha > \underline{a}$, and $E(a \mid a \geq \alpha) > E(a)$. Since $N - U > 0$, we have $(N-U)E(a) < (N-U)E(a \mid a \geq \alpha)$, or $NE(a) - (N-U)E(a \mid a \geq \alpha) < UE(a)$. Dividing both terms by U, we get that $w_U < E(a)$.

(iii) To prove this part I need to study the sign of $\frac{\partial w_U(U)}{\partial U}$. However, I know that this derivative can be written as:

$$\frac{\partial w_U(U)}{\partial U} = \frac{\partial w_U}{\partial \alpha}\frac{\partial \alpha}{\partial U}.$$

Since $\frac{\partial \alpha}{\partial U} > 0$, the sign of $\frac{\partial w_U(U)}{\partial U}$ coincides with the sign of $\frac{\partial w_U}{\partial \alpha}$. Computing the derivative of $w_U(\alpha; p)$ with respect to α I have:

$$\frac{\partial w_U(\alpha;p)}{\partial \alpha} = \frac{pg(\alpha)}{(1-p(1-G(\alpha)))^2}\left[\alpha + p(1-G(\alpha))\left[E(a \mid a \geq \alpha) - \alpha\right] - E(a)\right]. \quad (39)$$

Since $g(a) > 0$, the sign of the derivative will be the sign of the term in brackets. First I check what happens at the extreme points:

(1) $\alpha = \underline{a}$. The term in brackets becomes $(1-p)\left(\underline{a} - E(a)\right)$ which is negative, since $\underline{a} < E(a)$. This means that $\left.\frac{\partial w_U(\alpha;p)}{\partial \alpha}\right|_{\alpha=\underline{a}} < 0$. When $\alpha = \underline{a}$, $U = (1-p)N$, implying that $\left.\frac{\partial w_U(U)}{\partial U}\right|_{U=(1-p)N} < 0$ as well.

(2) $\alpha = \overline{a}$. The term in brackets becomes $\overline{a} - E(a)$ which is positive. In this case I have that $\left.\frac{\partial w_U(\alpha;p)}{\partial \alpha}\right|_{\alpha=\overline{a}} > 0$. When $\alpha = \overline{a}$, $U = N$, implying that $\left.\frac{\partial w_U(U)}{\partial U}\right|_{U=N} > 0$ as well.

Second, I check that the term in brackets in Equation (39) is an increasing function of α. To see this, just note that its derivative is $(1 - p(1-G(\alpha)))$, which is positive. Then there must be a unique $\widetilde{\alpha}$ at which $\frac{\partial w_U(\alpha;p)}{\partial \alpha}$ takes value zero. This value $\widetilde{\alpha}$ is the one that satisfies the equation:

$$\widetilde{\alpha} + p(1-G(\widetilde{\alpha}))\left[E(a \mid a \geq \widetilde{\alpha}) - \widetilde{\alpha}\right] = E(a). \quad (40)$$

Then $\frac{\partial w_U(\alpha;p)}{\partial \alpha}$ is a strictly decreasing function of α in the interval $[\underline{a}, \widetilde{\alpha})$, and it is a strictly increasing function in the interval $(\widetilde{\alpha}, \overline{a}]$. Accordingly, we have that $w_U(U)$ is a strictly decreasing function from $(1-p)N$ up to $\widetilde{U}$, and it is a strictly increasing function from $\widetilde{U}$ up to N, where $\widetilde{U}$ is related with $\widetilde{\alpha}$ through the expression $\widetilde{U} = N\left[1 - p(1 - G(\widetilde{\alpha})\right]$. ■

Proof of Proposition 3

I check first Condition (26). This condition can be expressed as $p_M a_M \leq \beta(p_M + p_L)a_H - K(p_M + p_L) - p_L a_L$. Since $p_M a_M < \beta p_M a_H$, it is enough to prove that $\beta p_M a_H \leq \beta a_H(p_M + p_L) - K(p_M + p_L) - p_L a_L$, or that $\beta a_H - a_L \geq K\frac{(p_M+p_L)}{p_L}$. But this is implied by Assumption 1 and the fact that $p_L \geq p_H$. To see this, note that I have to prove that:

$$(c + K)\left(\frac{p_H + p_M}{p_H}\right) \geq K\frac{(p_M + p_L)}{p_L}. \tag{41}$$

The worst case is when $c = 0$, in which case I need to prove that:

$$\frac{p_H + p_M}{p_H} \geq \frac{p_M + p_L}{p_L}, \tag{42}$$

which can be written as $p_L \geq p_H$.

To prove Condition (31), I can rewrite it as $\beta p_M a_M \geq (p_H+p_M)(a_L+c+K)-\beta p_H a_H$. Since $\beta p_M a_M \geq p_M a_L$, I just need to prove that $p_M a_L \geq (p_H + p_M)(a_L + c + K) - \beta p_H a_H$. But rearranging, this is just Assumption 1. ■

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