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Strategic behavior and information transmission in a stylized (so-called *Chinos*) guessing game

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ABSTRACT: A guessing game very popular in some European countries involves several players hiding in their hands a number of coins (or pebbles) between zero and three, then attempting to guess in turn the *total* number of coins in the hands of everyone, with the restriction that no player can repeat the guess issued by any predecessor. After a full round, the player, if any, who guesses correctly wins. Of course, rounds without a winner are also possible, in which case a new round is started afresh. The purpose of the present article is to present an analysis of this game (called *Chinos* in Spain, as a perturbation of "chinas", i.e. pebbles), and some of its possible variants. Our primary aim is to show its potential to shed light on some issues of strategic behavior and information transmission that seem very germane to some social and economic problems.
KEYWORDS: Information transmission, herding, Guessing game, Chinos game.

1. Introduction

As explained, *Chinos* is a simple non-cooperative game in which every player wishes to maximize her chances of guessing correctly and, consequently, minimize the guessing chances of her opponents. While in the actual game, the fraction of rounds without winner is an irrelevant consideration *per se* (apart from the time required to finish the game), it can in fact be an interesting variable to study in other contexts. Suppose, for instance, that we use the Chinos Game as a stylized metaphor of a financial market. That is, we conceive a setup where every agent has some private observation that is only very partially relevant to the "guessing contest" – say, anticipating the successful asset to hold – but can observe the decisions in this respect taken by those who moved first – i.e. those who chose their investment decisions earlier. In this context (and with this financial interpretation), a host of important and intriguing issues come to mind. What is the probability that someone makes the right (market) guess? What are the chances of herding behavior to arise? And what are the effects of this latter phenomenon on the informational efficiency of the market? Or more specifically, what are its effects on the ability of a large market in pooling effectively agents' dispersed information?

Of course, these issues have concerned economists and social scientists in general for a long time, and have been the motivation for a rather large body of literature. For example, herd behavior (or informational cascades) have been analyzed in a theoretical (abstract) setup in the seminal papers of Banerjee (1992) or Bikhchandani *et al.* (1992). And turning to the sphere of applications, one finds a very diverse crowd. Thus, for example, Anderson and Holt (1997) explore these phenomena in laboratory experiments, Kennedy (1997) or Chaudhuri *et al.* (1997) focus on how firms shape their business strategy, Welch (1992) studies consumer behavior, Glaeser *et al.* (1996) or Kahan (1997) deal with the spread of crime, and Lohmann (1994) with political action.

This approach has also been used to study financial markets, which we want to propose as our leading motivation. The reader is referred to the survey article by Camerer (1989) or the more recent work of Avery and Zemsky (1998). While this research is motivated by a similar concern, we want to argue that the Chinos Game affords a specially rich and transparent scenario to understand the interplay of strategic behavior and information transmission along a sequential decision process. In this game, the tension between the benefit of strategic preemption (when being an early mover) and the risk of informational leakage (that can be enjoyed by late-comers) arises very sharply. On the other hand, the potential for herding behavior to generate perverse informational cascades and consequent informational inefficiencies is quite apparent as well. However, we should also stress that the model is a very imperfect model of market environments in that it allows no role for (endogenous) price formation. And, as the work of Avery and Zemsky (1998) illustrates, endogenous price movements can sometimes offset inefficient herding in markets. An introduction of these richer considerations is left for future research.

2. The Model

As a well-defined game, *Chinos* may be analyzed with the standard equilibrium tools of classical Game Theory. In particular, we could aim at characterizing its Nash (or Subgame-Perfect) equilibria. Even though this can be readily done in simple cases (e.g. with up to one coin per agent, as in Guinea (2000), or just two players, as in Weibull (2000)), the analysis for the general case is quite complicated. Thus, to gain some first insight on the game, we choose to simplify matters by focusing on a simplified version of it where (a) the coins in the hand of each player are chosen by "Nature", randomly (and independently) for each player; (b) the players are not precluded from repeating the same guess as their predecessors. As it turns out, this version of the game allows for a simple and clear-cut analysis, which we shall later use as a basis for our approach to the genuine Chinos game (i.e. the version of it where coincidence of guesses is not permitted).

To be precise, we shall first consider the following game. Players are indexed by $i = 1, 2, ...N$ according to their order of play. A priori, each of them can hold up to m coins, i.e. the number of coins ξ_i held by each player i is determined by a uniform random variable chosen with support on the set $\{0, 1, ..., m\}$. Each player i is informed of her respective ξ_i but only knows that others will have their number coins in the described fashion.

Let x_i denote the guess of player i. Then, by a simple inductive reasoning, it is easy to verify that, given the realized vector of coins $\xi = (\xi_1, ..., \xi_N)$ that happen to lie in the hands of the the N players, the corresponding vector of guesses at the unique *Nash equilibrium* (NE) of the induced game is given by the following simple formula:

$$x_i = (x_{i-1} - \mu_0) + (\mu_1 + \xi_i) \qquad (i = 1, 2, ..., N) \tag{1}$$

where $\mu_0 \equiv Nm/2$ is the expected total number of coins by the whole population, $\mu_1 \equiv (N-1)m/2$ is the the expected total number of coins in the hands of $N-1$ players, and x_{i-1} is simply identified with μ_0 for $i = 0$. The interpretation of Eq. (1) is both straightforward and appealing:

1 On the one hand, the first term accounts for the information passed to player i by player $i-1$. This term alone embodies *sufficient* information

(at equilibrium!) on the extent to which the guesses of all preceding players deviate from the "average" uniformed guess. In that sense, we will speak of x_{i-1} (or equivalently, $x_{i-1} - \mu_0$) as the *state* of the process at stage i and label the above rule *Markovian*. Note, specifically, a powerful implication of this fact: by observing x_{i-1}, player i is able to gain *precise* knowledge of the number of coins $\sum_{j<i} \xi_j$ in the hands of all predecessors.

2 Then, complementing that information revealed thus far, the second term of (1) adds the term $\mu_1 + \xi_i$ which simply reflects what player i can deduce from her exclusive (private) information. For, if she only had that private information, player i would simply add her number of coins ξ_i to the expected number μ_1 held by the other $N-1$ players.

Clearly, it follows from the above construction that, at her time of choice, player N is fully informed and thus is sure to guess correctly (perhaps not alone). Thus, in this simplified version, the equilibrium always involves full revelation, in the sense that at least one individual is certain to guess correctly. In general, the probability P_i that any particular player i guess correctly must increase as i approaches N. For large N, the central limit theorem (see, for instance, Sobol (1994)) gives the following result for this probability (cf. the Appendix)

$$P_i = \sqrt{\frac{6}{m(m+2)\pi}}\frac{1}{\sqrt{N-i}} \tag{2}$$

where m is the maximum number of coins a player can hide in her hands. This expression, which is only valid when i is not too close to the end of the game, is in excellent agreement with the numerical results for $m = 2$ shown in Figure 1.

At this point, it is worth emphasizing the obvious: the behavioral rule given by (1) is only an optimal course of action if, and generally only if, each player i can be confident that the players $j < i$ follow the analogous rule as well. This feature, which is in essence a mere re-statement of the NE stability condition, has particularly forceful implications in the present context. If any given player i harbors any doubt that others may react according to the equilibrium prescriptions (either because, say, they do not understand the situation or are just *noise agents* with other considerations in mind), then she may be inclined to use a richer rule that generalizes (1) in the following two respects:

(a) all preceding observations are heeded (i.e. the rule is no longer Markovian, in the above mentioned sense)
(b) past observations, recent and not, are weighed in some discretionary manner.

A generalized rule of this kind is as follows:

$$x_i = \sum_{j<i} \lambda_{i,j}(x_j - \mu_0) + (\mu_1 + \xi_i) \tag{3}$$

where $\lambda_{i,j}$ are real numbers (either positive or negative) that reflect the sensitivity of player i to player j's (prior) guess. Note that (1) follows from (3) as a particular case if $\lambda_{i,j} = 1$ if $j = i - 1$ and zero otherwise.

Let us now return to the original game of *Chinos* where, as the reader will recall, no two players are allowed to guess the same number. This game is interesting in itself because it captures the aforementioned tension between strategic preemption and information leakage in ways that the simplified version cannot do – in the latter version, only the second consideration is present. However, as explained, the original game is so much more complex to analyze that it is difficult to see how one could obtain a closed analytical solution for its Nash equilibrium.† Therefore, we have chosen to approach it through numerical simulations, by building upon the general behavioral format given in (3) that is suggested by our previous analysis of the simplified version. Furthermore, we choose to narrow down the number of different parameters involved by making

$$\lambda_{i,j} = \frac{\tau_i}{\mid x_j - \mu_0 \mid}. \tag{4}$$

This amounts to having each player i add (or subtract) a parameter τ_i if the guess of her predecessor j is larger (or smaller) than average, regardless of the size of the deviation. In much of our discussion, all players are posited either to display an identical τ_i or be divided into two homogeneous subsets.

3. Simulations and Results

Numerical simulations for an arbitrary number of players with $m = 3$ coins and strategies defined by Eq. (4) have been carried out. We have also introduced what we call a *noise agent* (NA), a kind of player who happily guesses random numbers with complete disregard of statistics or previous players guesses. The only restriction a noise agent is subject to is that she should not break the rules of the game, as stated at the beginning of the paper. The results are illustrated in figures 2–4.

Figure 2 shows the percentage of games won by each player from a set of 10 players. When $\tau = 0$, every player disregards the information from previous players. The percentage of games won by a given player smoothly decreases as the position of the player moves towards the end of the round. This is because, toward the end of the round, good guesses are already taken and the player is forced to guess far from the statistic average. When $\tau = 0.7$, the curve shows a maximum at intermediate positions. The reason is that intermediate players may use the information transmitted by previous players to make a better guess. However, this information again becomes useless towards the end of the round and therefore the percentage of games won drops steadily. For $\tau = 1.5$, the curve is similar, although now the maximum occurs at the second position, the drop after that being very fast and leading to a very low minimum. To understand why, we should remember that in our model, if a player guesses over (under) the average, no matter how small this overestimation (underestimation) is, it is assumed that she carries τ more (less) coins than the average of 1.5 per player. As soon as one player holds a number of coins above (below) the average, and guesses accordingly, she leads all the subsequent players astray, each one assuming that

† Nevertheless, some such equilibrium exists, since the game is finite. But, of course, the same applies to the game of Chess, even though the problem is computationally unmaneagable.

all the previous ones hold τ more coins than average. Not surprisingly, the odds of the players at the end of the round are very low, not because the good guesses are already taken, as was the case for $\tau = 0$ and $\tau = 0.7$, but because the sequence of guesses hopelessly drifts away from the statistic average (*herd behavior* (Banerjee, 1992)). Finally, we will deal with $\tau = -1.5$, which corresponds to the case of "distrustful" players who think that, even if a player guesses above average, in reality she holds a number of coins below average but she is trying to fool others. The players, accordingly, subtract (add) 1.5 whenever they detect overestimation (underestimation). This results in an oscillatory sequence of guesses, because if a given player guesses over the average, then the next one will subtract -1.5 and therefore will probably guess under the average, so that the next player will be forced to guess over the average, and so on.

The percentage of lost rounds (that is, those rounds for which no player was able to guess successfully) is presented in figure 3 as a function of the total number of players, when all of them share the same $\tau_i = \tau$. With the aim of providing a useful benchmark, the case where all players are NA is also shown. The curves in this figure can be easily understood after the previous discussion on figure 2. Basically, they can be divided into two classes: either the percentage of lost games decreases as the number of players grows large or it increases. The first class ($\tau = 0$, $\tau = 0.5$, or $\tau = -1.5$) can be explained by the absence of herd behavior: players' guesses are reasonably close to the average, so that sooner o later some player ends up guessing the total number of coins, the greater the number of players the more likely. The shape of the second kind of curves ($\tau = 1.0$ or $\tau = 1.5$) is caused by herd behavior: as soon as a player gives a guess far enough from average, all subsequent players are led astray and the round will finish with no one guessing the right number of coins. Only the first players will have a fair shot at guessing correctly before herd behavior sets in but, since the statistical distribution of possible results widens, their odds will become smaller as the total number of players increases. Finally, the curve with $\tau = 0.7$ shows a hybrid behavior, first decreasing to almost zero, then increasing.

Figure 4 shows the percentage of lost games for a mixture of two kinds of players, "trustful" ($\tau = 0.7$) and "distrustful" ($\tau = -0.7$), and compares it to the "pure" counterparts. It is remarkable that the presence of distrustful players keeps the mixture in check and prevents herd behavior from appearing. On the other hand, the fraction of trustful players makes the mixture more successful than a pure set of distrustful players.

Next, we wish to investigate how the presence of a *single* NA distorts the results discussed above. Of course her effect strongly depends on its position, being stronger when she plays in first place. Figs. 5 and 6 show results of simulations similar to those of Figs. 2 and 3 but with a NA in the first position. Comparing the two sets of figures leads to the conclusion that trustful strategies are very sensitive to perturbations, while distrustful playing is more robust. Of course the less sensitive strategy is that which does not take into account the information from previous players ($\tau = 0$). Particularly remarkable is the early appearance of *herd behavior* when players follow trustful strategies ($\tau > 0$). This is clearly illustrated by the results for $\tau = 0.7$ of Fig. 5. On the other hand, concerning the percentage of lost rounds, Fig. 6 shows that for $\tau = 1.5$ it equals that obtained with all players being NA for N approximately greater than 10. It is worth noting that large positive τ may lead to a percentage of lost guess higher than that obtained with all players being NA.

As has been said concerning Eq. (4), a given player, when computing her guess, adds (or subtracts) a *constant* amount τ for each previous player that guesses above (below) the average. By the central limit theorem, the sum of all contributions τ for player i is a normal distribution with a standard distribution proportional to $\sqrt{i-1}$. Accordingly, players located at the end of the round (high i) will probably guess far from average and suffer from herd behavior. In an attempt to eliminate herd behavior, we introduce a new kind of player, the *normalized agent*, for which the τ_i that appears in Eq. (4) is set to the value

$$\tau_i = \frac{\tau}{\sqrt{i-1}}. \tag{5}$$

With this definition for τ_i, as can be seen in figures 7 and 8, normalized agents not only do not show herd behavior, but give an excellent overall performance.

4. Conclusions

This paper has focused on a stylized guessing game (i.e. Chinos) where the standard herd effects (or informational cascades) often displayed by sequential models of informational transmition are significantly enriched by strategic (i.e. game-theoretic) considerations. Our analysis has been mainly concerned with three issues. On the one hand, we have been interested in understanding how the "pre-emption advantages" of playing early in the game are offset by the opposing "informational benefits" of later playing. The optimal compromise between these two countervailing forces has been shown to depend quite intuitively on the key parameter of our model modulating players' responsiveness to the guesses put forward by earlier movers. Secondly, we have studied how such a responsiveness affects the likelihood that the right number of coins be guessed or, analogously, that virtuous or perverse information cascades should obtain. Finally, we have addressed natural issues of robustness, the question then being how our previous insights are affected by the intrusion of an early noise player that could trigger an accumulating sequence of distorted guesses. We believe that the game of Chinos may represent a convenient benchmark to gain some useful insights on the interplay between information transmission and strategic play in sequential games. It has the modeling advantage of being specially simple but still rich enough to permit posing some of the key questions which are bound to appear in a more involved fashion in other more directly applicable setups, e.g. models on the formation of consumer preferences, financial trading, or the spread of political ideas. Quite interestingly, it also bears some relationship to a game (Steane and van Dam, 2000) recently used by physicists in the rapidly growing fields of quantum computing and quantum information theory.

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A. Appendix

Here we derive the probability that a particular player in the simplified version of the *Chinos* game (with repetition) guesses correctly. Players can hide in their hands a number of coins in the range $0, 1, \ldots, m$ with equal probability $1/(m+1)$. This is a random variable which we call ξ. The average or expected value M_ξ of this random variable and its variance D_ξ are given by,

$$M_\xi = \frac{m}{2}, \qquad D_\xi = \frac{m(m+2)}{12} \ . \tag{A.1}$$

In the *Chinos* version that allows repetition, player i knows what her predecessors hide in their hands. Then, her chances of guessing correctly are completely determined by the probability of guessing correctly the total number of coins that the players guessing after her hide in their hands. Then, if the player is sufficiently far from the end of the game or, in other words, $N - i >> 1$, where N is the total number of players, this probability can be derived from the Central Limit Theorem (CLT). Calling ξ_j the random variable related to the coins hidden by players after i, we define a new random variable that gives the total number of coins that those players hide, as,

$$\rho_{N-i} = \sum_{j=i+1,\ldots,N} \xi_j \ . \tag{A.2}$$

This random variable can take the values $r = 0, 1, 2, \ldots, m(N-i)$. If the number of terms in this sum is sufficiently large, what the CLT states is that the random variable ρ_{N-i} is normal with expected value a and standard deviation σ given by

$$a = \frac{m(N-i)}{2}, \qquad \sigma = \sqrt{(N-i)D_\xi} \ . \tag{A.3}$$

Now we have to calculate the probability that the actual number of coins coincides with $m(N-i)/2$, which is the guess of player i for the number of coins that players guessing after her hide in their hands (see main text). The probability distribution of the random variable ρ_{N-i} is a continuum function only for infinite $N-i$. For smaller values of this parameter it is composed of delta functions modulated by a Gaussian function. Then, the probability we are seeking coincides with the maximum of the normal probability density $p(a) = 1/(\sigma\sqrt{2\pi})$, namely,

$$P_i = \frac{1}{\sigma\sqrt{2\pi}} = \sqrt{\frac{6}{m(m+2)\pi}} \frac{1}{\sqrt{N-i}} \ . \tag{A.4}$$

This result is in excellent agreement with the numerical results discussed in the text.

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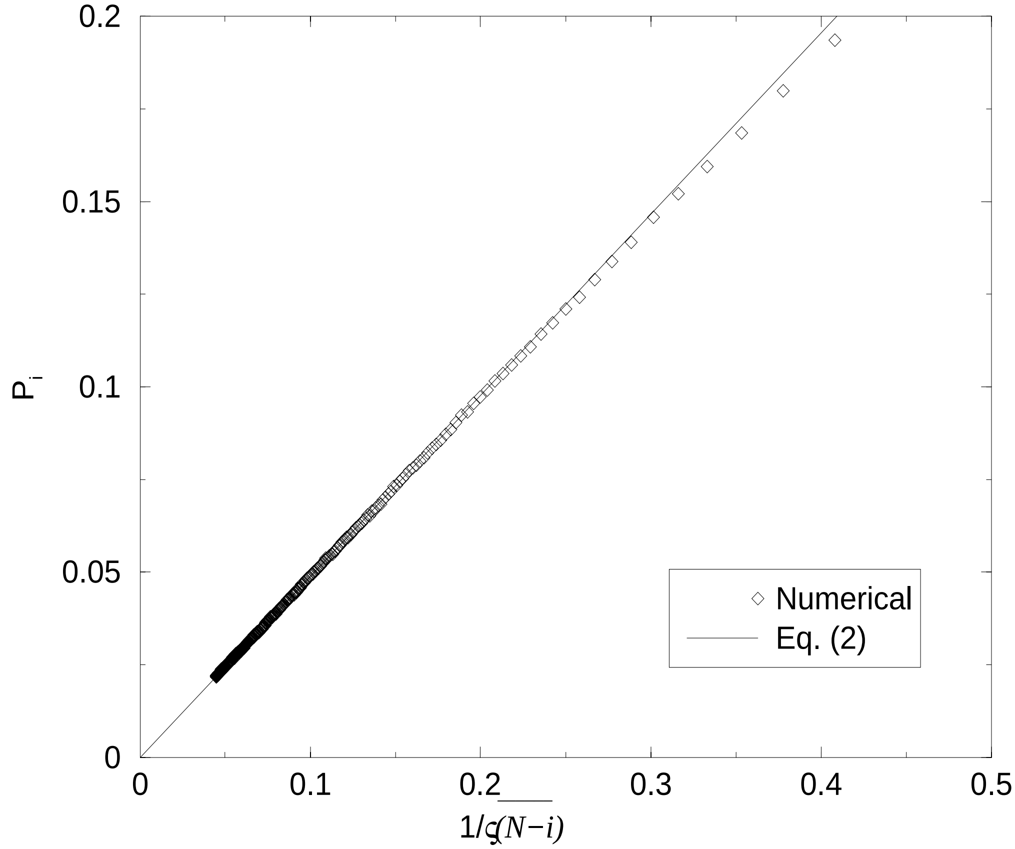

Figure 1: Numerical verification of Eq. (2) for a set of N=500 players, where i denotes the position of a given player.

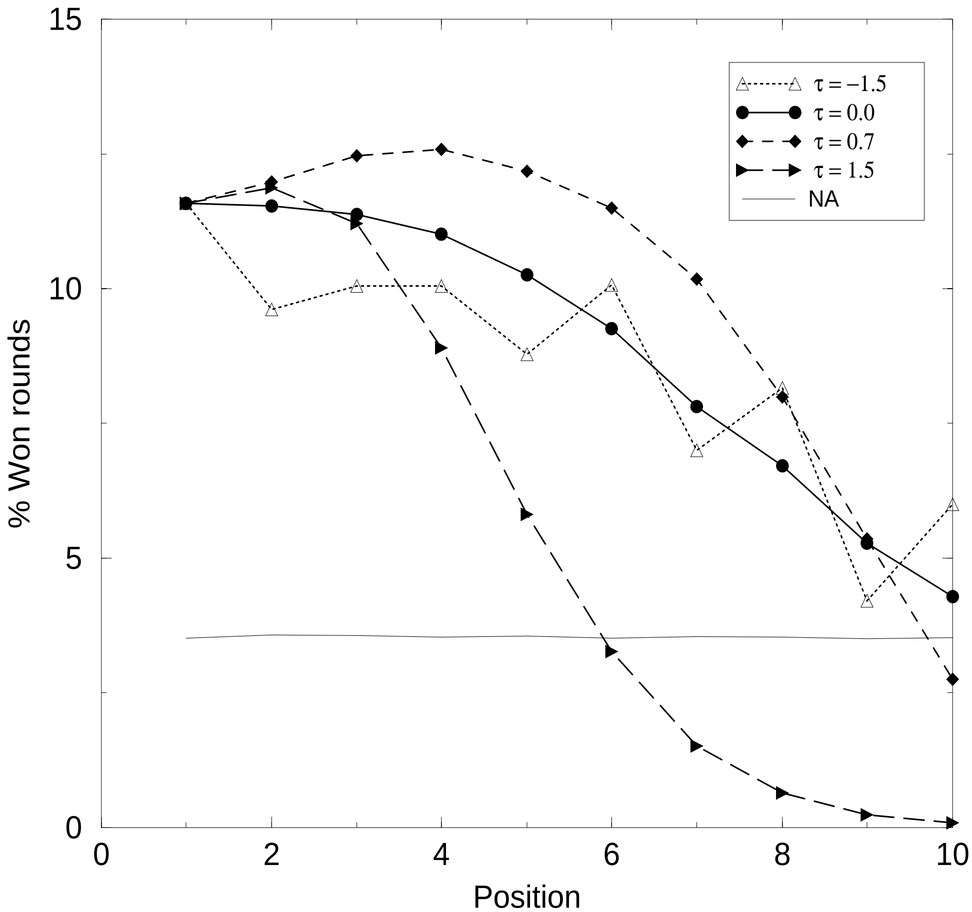

Figure 2: Percentage of rounds won by a given player vs. the position of the player in the round. The simulation is carried out as follows. First, a random number of coins with a uniform probability distribution between 0 and m is assigned to each player. Second, each player tries to guess the total number of coins in turn, according to she respective strategy, either Eq. (4), or the noise agent strategy. Third, if the guess chosen by the player is already taken, then she selects the nearest free guess. A lot of rounds (half million) are played in this way, and the average is computed and shown in the figures.

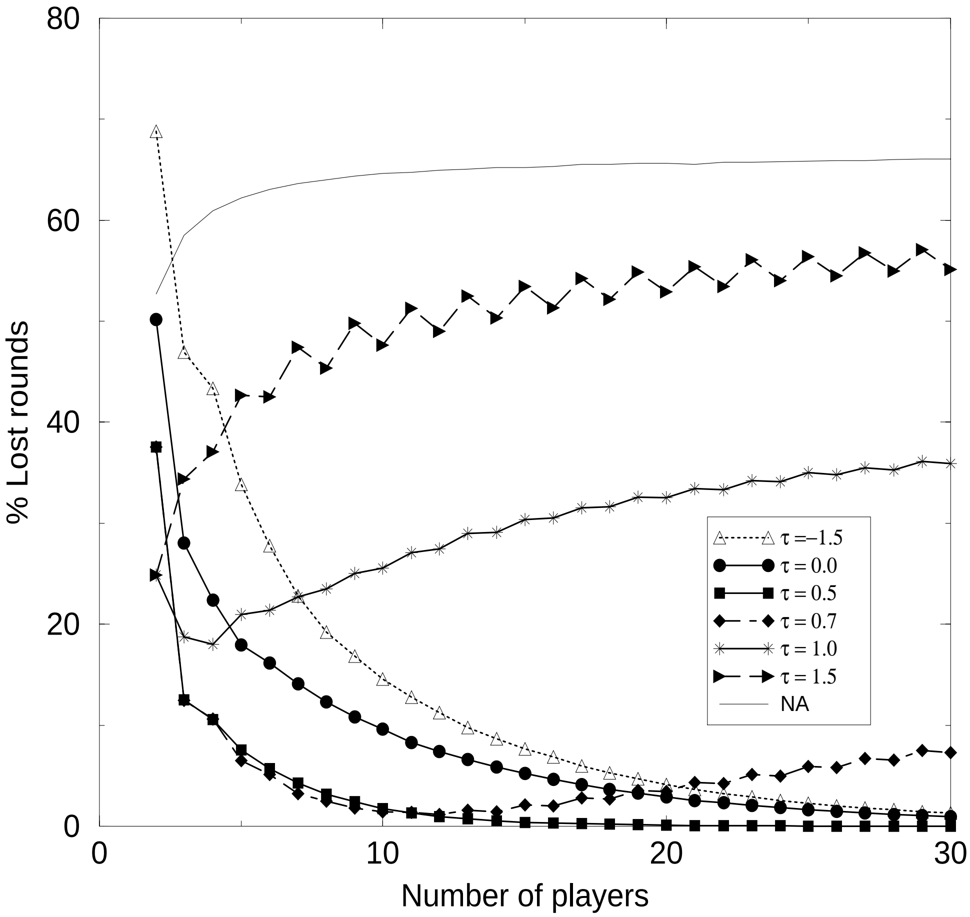

Figure 3: Percentage of lost rounds vs. total number of players N for sets of players with uniform $\tau_i = \tau$. The results obtained when all players are noise agents (see text) are also shown (continuous line); in this case the percentage of lost rounds can be calculated analytically, the result being $P_{\mathrm{lost}} = 100(N-1)/((m/(m-1))N-1)$, where m is the maximum number of coins hidden by each player.

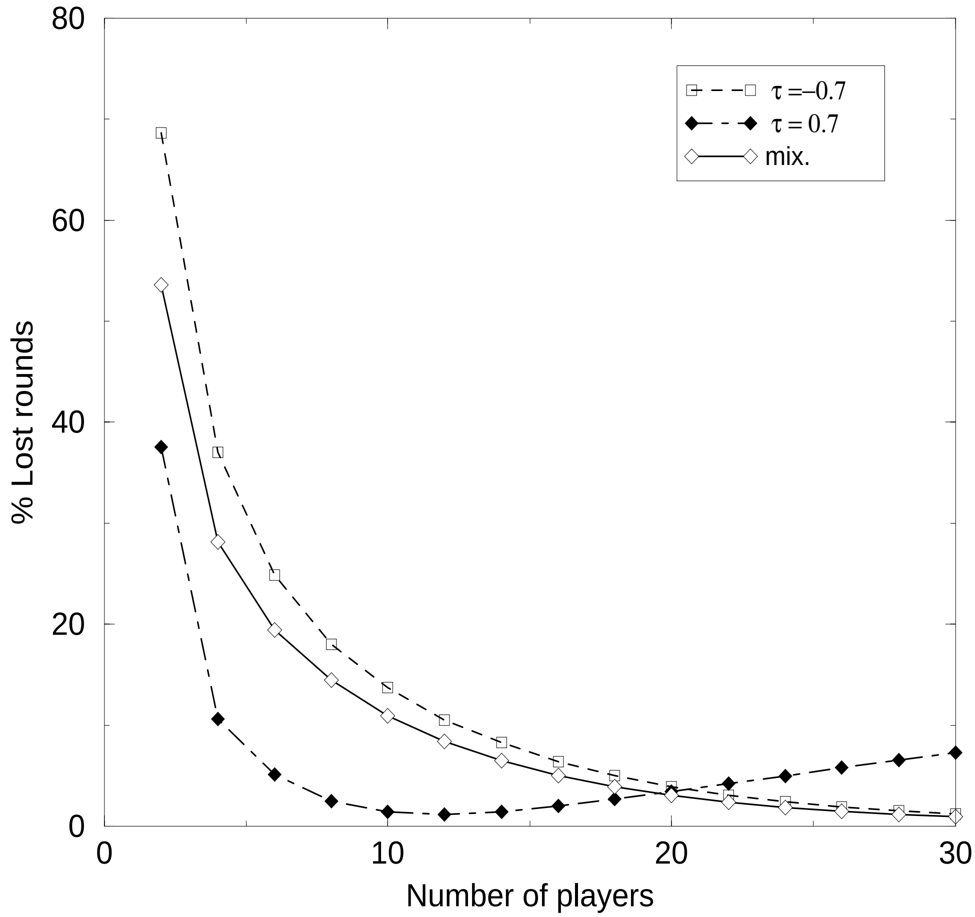

Figure 4: Percentage of lost rounds vs. total number of players for a 50%–50% mixture of "trustful" ($\tau = 0.7$) and "distrustful" players ($\tau = -0.7$). "Pure" cases are included as references.

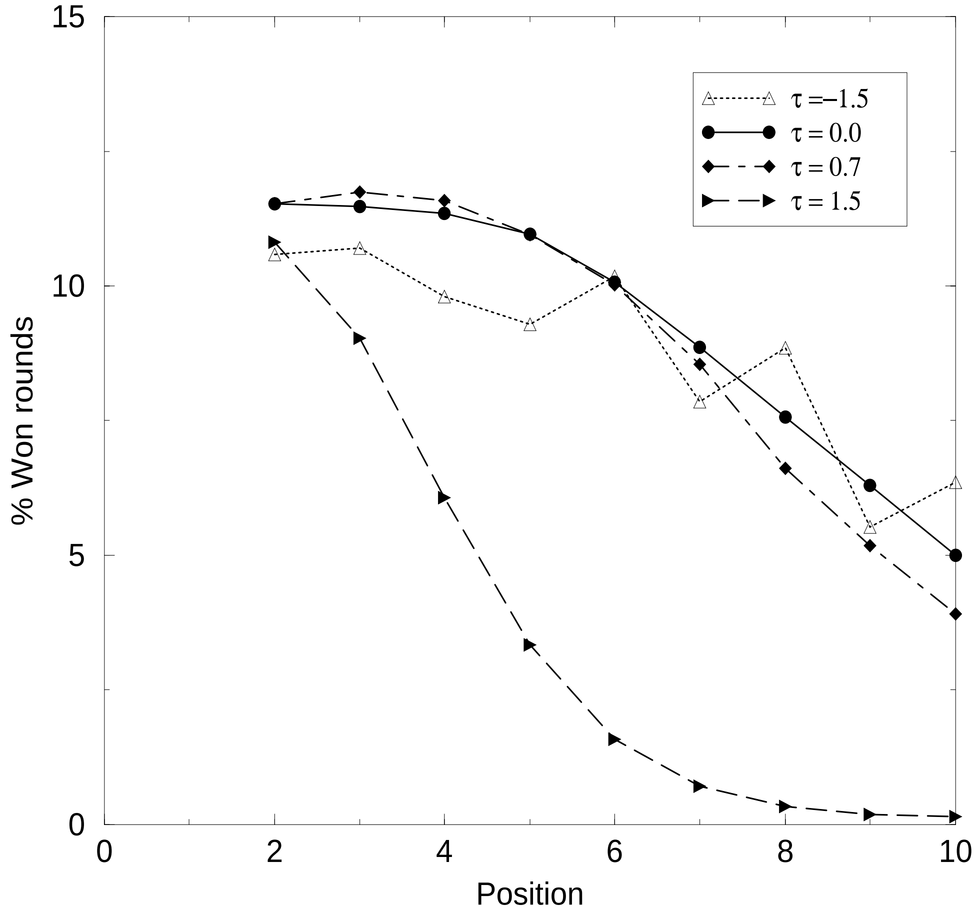

Figure 5: Same as Fig. 2 with a *noise agent* in the first position.

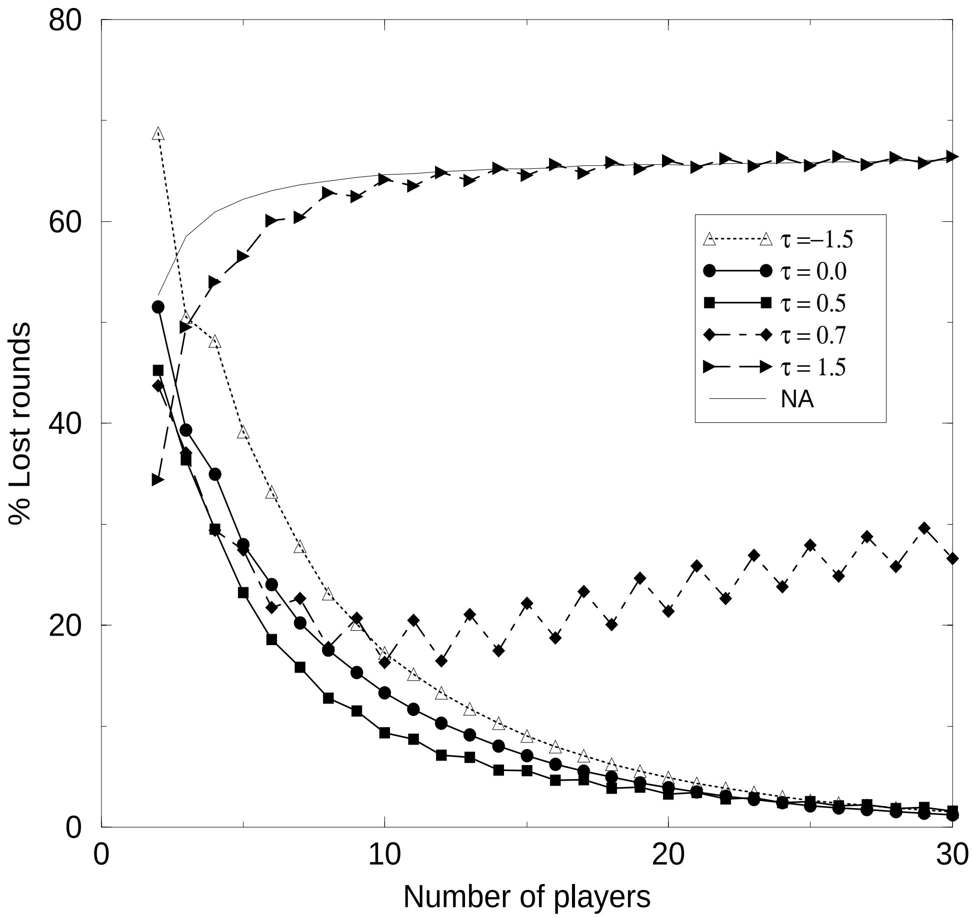

Figure 6: Same as Fig. 3 with a *noise agent* in the first position.

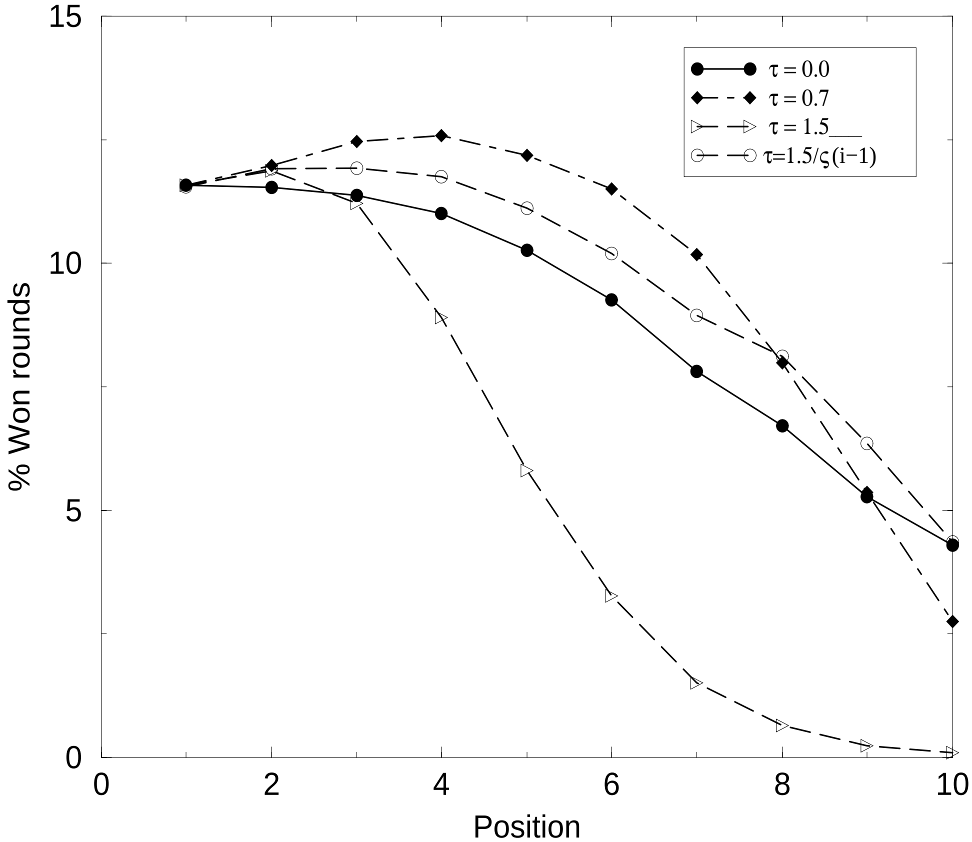

Figure 7: Same as figure 2, but including an additional curve for *normalized agents*.

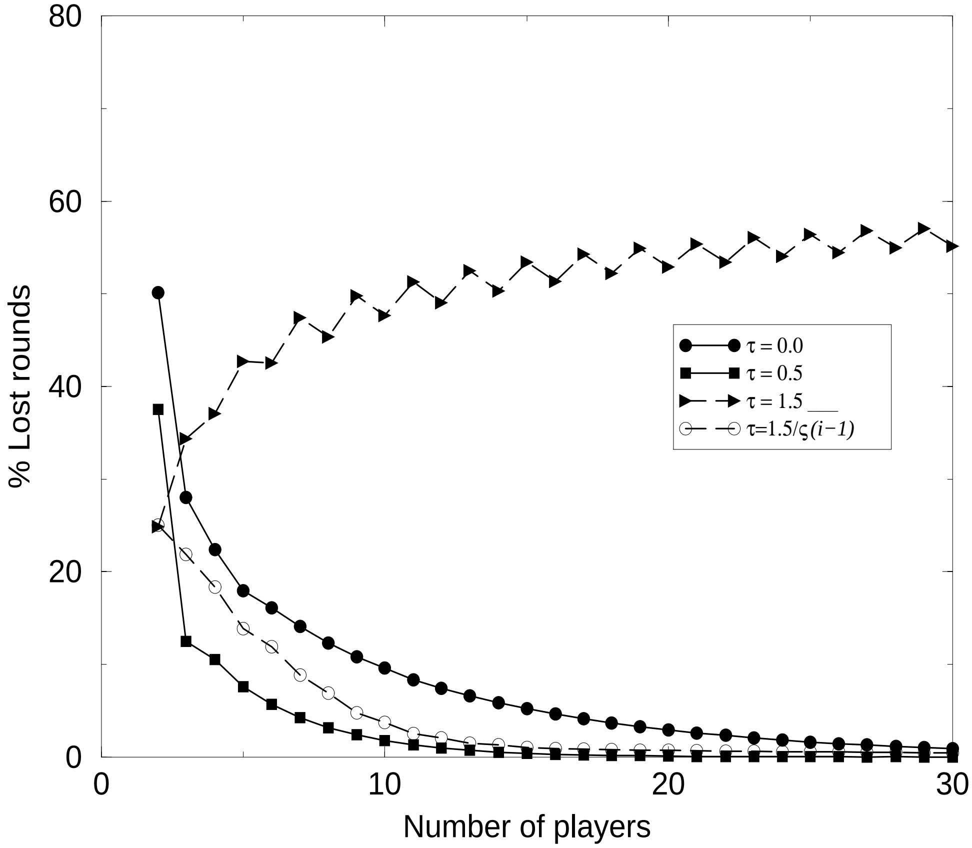

Figure 8: Same as figure 3, but including an additional curve for *normalized agents*.