Imperfect Learning and Error Cascades in Sequential Guessing Games: An Experiment*

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Abstract

This paper reports on an experiment designed to analyze information revelation in sequential guessing games. We observe a remarkable frequency of equilibrium outcomes. Deviations from the equilibrium are explained by learning considerations. By means of applying panel data econometric techniques, we find the presence of error cascades, which represent the situation where the deviation from learning by a player increases with respect to deviations from learning by preceding players.

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1 Introduction

There are situations where agents have to take public decisions sequentially under uncertainty. Consider for example the case of financial markets or the choices of firms on technological adoptions under uncertain market conditions. In these situations agents may have private but incomplete information about which the profitable decision is. Hence, the higher the number of agents who have taken their decisions, the lower the level of uncertainty faced by those who still have to take their action. Nevertheless, even if later players in the sequence have less uncertainty, their problem is to decode predecessors' private information from the observed actions. In many of these situations former players in the sequence have the advantage of strategic preemption, but in this paper we want to focus on analyzing whether later players are able to take advantage of their position to have a higher chance to take the correct action.

Therefore in this paper we design an experiment want to analyze agents' behavior in signaling games with no strategic conflict among agents. We consider a situation where there is a sequence of players who have to guess which the true state of the world is. Each of these agents have private information about the true state of the world and they make their guess publicly and sequentially. Hence throughout the guessing sequence private information may be revealed to later players who can use this fact to take the correct action with a higher probability.

Our focus is on a very simple strategic context, which concerns a traditional parlour game played in many countries, which in Spain is known as *Chinos*.[1] In this game, players start by hiding in their hands a number of coins (or pebbles), from zero to a certain maximum number (often three). Then, in some pre-specified order, each player produces a guess on the *total* number of coins in the hands of every player. When doing so, a player is informed of her own number of coins and, if she is not the first one to speak, the guesses produced by all others who preceded her.

Formally, the game is to be conceived as a multi-stage game with incomplete information. In our version of the game, it is assumed that the number of coins in the hands of each player is the outcome of an exogenous random mechanism – i.e. a stochastic choice by Nature. We further simplify matters

[1] The word "chinos" is a slight modification of the Spanish word "chinas", which refers to the pebbles that players may hide in their hands when playing the game.

by considering just three players and restricting the number of coins in the hands of each player to be either zero or one. Finally, concerning payoffs, we design the game so that players' incentives do not conflict. Therefore, their decisions should not reflect any strategic considerations and thus represent a perfect signal of their private information. In this case, there is a unique Perfect Bayesian Equilibrium (PBE) where, after observing each player's guess, any subsequent players may infer exactly the pebbles lying in the formers' hands, and, thus, the last player in the sequence guess the correct answer with certainty.

In this light, the main objective of our experiments can now be advanced. Succinctly expressed, it is to contrast whether, as theory would unambiguously prescribe, early moving subjects choose clear-cut signalling guesses *and* later players are able to "decipher" them and act accordingly. These are the main regularities we would expect to find in the experimental evidence, possibly entangled by considerations of learning and noise, unavoidable in any real-world context.

We find that, qualitatively, the experimental results reproduce the theoretical predictions. First, if we consider the outcomes we observe that the frequency with which the correct answer is guessed is increasing in the player position. Hence, later players in the sequence are able to use the information they infer from predecessors to have a higher chance to guess right. Nevertheless we find players make errors, since these frequencies are lower than it is predicted by theory. In fact the higher the player position the higher the difference between frequency of actual play and equilibrium prediction is. Second, if we analyze the behavior of agents we find a high frequency of equilibrium play, i.e. modal play coincides with equilibrium play, but we also find deviations from equilibrium.

Hence, the equilibrium analysis seem to explain the data, but only imperfectly: there are deviations from equilibrium which we rationalize by doing a deeper econometric analysis. We argue that deviations from equilibrium are due to the way players learn to play. In our experimental sessions we keep groups fixed to favor agents' learning and in order to understand agent's play with respect to their observed history.[2] Additionally, the use of fixed groups allows us to have a higher number of independent observations and the pos-

[2]Note that in our framework there should not be repeated game effects, since there is a unique equilibrium of the stage game. Hence, the unique equilibrium of the entire game is the repetition of the equilibrium of the stage game. Moreover, the fact that there is no strategic conflict among agents also prevents the possibility of repeated game effects.

sibility to make a panel data econometric analysis. This analysis represents a novelty in the experimental literature since it allows us to interpret agents' errors with regard to their own experience, instead of assuming that errors are i.i.d. We say there is learning when players are able to react optimally to their past experience, i.e. to the strategy they infer their predecessors are playing. Our estimations show the presence of imperfect learning: a player's learning is less intense the higher the deviation from optimal behavior by his predecessors. Hence, deviations from the equilibrium are explained by the accumulation of errors: the presence of imperfect learning generates what we call an "error cascade", i.e. the situation where deviation from learning by a player increases with respect to the deviations from learning by preceding players. We show the presence of error cascades, which allows us to explain the experimental results relying on learning arguments.

The *Chinos* game was first strategically analyzed by Pastor-Abia et *al.* (2000), where they analyze a generalized version of the game. The issue of information transmission has been the motivation for a rather large body of literature. For example, herd behavior and information cascades has been analyzed in a theoretical setup in the seminal papers of Banerjee (1992) or Bikhchandani et *al.* (1992). And turning to the sphere of applications, one finds a very diverse crowd. Thus, for example, Kennedy (1997) or Chaudhuri et *al.* (1997) focus on how firms shape their business strategy, Welch (1992) studies consumer behavior, Glaeser et *al.* (1996) or Kahan (1997) deal with spread of crime and Lohmann (1994) with political action. This approach has also been used to study financial markets. The reader is referred to the survey article by Camerer (1989) or the more recent work of Avery and Zemsky (1998). This theoretical literature has also been object of experimental studies, among which we highlight Anderson and Holt (1997), who check in the lab the model of Bikhchandani et *al.* (1992). There has been a branch of experimental papers which have been framed in the setup of Anderson and Holt (1997) to analyze various interesting aspects of the problem at hand, among which we highlight Hung and Plott (2001), Kübler and Weizsäcker (2003) and Çelen and Kariv (2001a)-(2001b). This experimental literature is commented in detail after we present the results of our experiments.

The remainder of the paper is organized as follows. The next section provides a brief synopsis of the theory underlying the experiment. Section 3 describes the experimental design and procedures. Section 4 presents the results of the experiment in summary statistics. Section 5 introduces an analysis of learning and section 6 shows the presence of error cascades. Section

7 reviews the related experimental literature. Finally, section 8 concludes.

2 The model

In the *chinos game* every player $i \in N = \{1, ..., n\}$ receives, as private information, a signal s_i drawn independently from a probability distribution over the set of possible signals $S = \{0, ..., r\}$. Players act in sequence and have to guess the sum of signals over the n players which we denote by $\alpha = s_1 + ... + s_n$. By the time player i makes a guess $g_i \in G \equiv \{0, ..., n \cdot r\}$, she knows her signal (s_i) and the guesses of those who acted before her in the sequence. This is, player 1 guesses first, knowing the realization of her own signal (s_1). Player 1's guess, g_1, is observable by the rest of the players. Then, player 2, having observed her own signal (s_2) and g_1, makes a guess, which is made public. Then, player 3 make a (public) guess knowing her signal (s_3) and the guesses made by players 1 and 2, and so on.

Now we define the payoff function, that is, how monetary rewards are distributed depending on players' signals and guesses: all players who guess correctly (i.e. $g_i = \alpha$) receive a fixed prize, which we normalize to 1. All players who do not guess correctly (i.e. $g_i \neq \alpha$) receive zero payoff. It is clear that, in this case, all players have an incentive to maximize their chances to guess correctly, regardless of the fact that their guesses will reveal their signals to other players.

By analogy with the experimental treatments, we characterize the equilibria under the following conditions. First, we focus on the case of three players ($n = 3$) and two signals ($r = 1$). Moreover, we shall assume signals s_i are independently drawn from the same probability distribution, with p denoting the probability of $s_i = 1$. Without loss of generality, we fix $p \geq \frac{1}{2}$.

Note that the sum of signals for k players will be the realization of a Binomial distribution $B(k, p)$. Let M_k be the mode of that distribution.

We shall look for Perfect Bayesian Equilibrium (PBE) of this game under the different payoff scenarios specified below. In what follows, we set the notation for players' behavioral strategies and beliefs.

Let $\mathcal{I}_i$ represent the set of possible information sets[3] of player i, *i.e.*

[3] An information set of player 1 consists on her own signal ($I_1 = s_1$), an information set of player 2 consists on her own signal and the guess made by player 1 ($I_2 = (s_2, g_1)$), and an information set of player 3 consists on her own signal and the guesses made by her

$\mathcal{I}_1 \equiv S$, $\mathcal{I}_2 \equiv S \times G$, and $\mathcal{I}_3 \equiv S \times G^2$, with I_i denoting a generic element of $\mathcal{I}_i$. In words an information set of player 1 consists on her own signal $(I_1 = s_1)$, an information set of player 2 consists on her own signal and the guess made by player 1 $(I_2 = (s_{2,}g_1))$, and an information set of player 3 consists on her own signal and the guesses made by her predecessors $(I_3 = (s_{3,}g_1, g_2))$.

The strategy of a player $i \in \{1, 2, 3\}$ is represented by $\gamma_i = \{g_i(I_i)\}_{I_i \in \mathcal{I}_i}$

We define the belief system of player i as $\{\mathcal{M}_i(I_i)\}_{I_i \in \mathcal{I}_i} \equiv \{\{\mu_i(I_i)(h)\}_{h \in I_i}\}_{I_i \in \mathcal{I}_i}$, where $\mu_i(I_i)(h)$ represents the probability that player i assigns to the history $h \in I_i$, conditional on the information set I_i being reached.

Formally:

$\mu_2(I_2)(h) = \Pr_2(h|I_2) = \Pr_2(s_1, s_2, g_1|s_2, g_1)$.

$\mu_3(I_3)(h) = \Pr_3(h|I_3) = \Pr_3(s_1, s_2, s_3, g_1, g_2|s_3, g_1, g_2)$

Hence, simplifying the above expressions, using the fact that s_1, s_2, s_3 are *i.i.d.*, we get:

$\mu_2(I_2)(h) = \Pr_2(s_1|s_2, g_1) = \Pr_{2,1}(s_1|g_1)$.

$\mu_3(I_3)(h) = \Pr_3(s_1, s_2|s_3, g_1, g_2) = \Pr_3(s_1, s_2|g_1, g_2) = \Pr_{3,1}(s_1|g_1, g_2) \cdot \Pr_{3,2}(s_2|g_1, g_2) = \Pr_{3,1}(s_1|g_1) \cdot \Pr_{3,2}(s_2|g_1, g_2)$

Thus, defining the subjective probabilities $\Pr_{2,1}(s_1|g_1)$, $\Pr_{3,1}(s_1|g_1)$ and $\Pr_{3,2}(s_2|g_1, g_2)$ we completely define a system of beliefs for all the population.

Hence, given the realized vector of signals (s_1, s_2, s_3), the corresponding vector of guesses at the unique PBE of the game will be as follows:

$$\begin{aligned} g_1 &= s_1 + M_2 \\ g_2 &= (g_1 - M_2) + s_2 + M_1 \\ g_3 &= (g_2 - M_1) + s_3 \end{aligned}$$

Note that M_1 and M_2 represent common information, hence player 2 and player 3 can infer s_1 from g_1 $(s_1 = g_1 - M_2)$. Thus player 3 can infer $s_2 + s_1$ from g_2 $(s_1 + s_2 = g_2 - M_1)$.

This is, in equilibrium, each player reveals her signal. Hence player 1 has uncertainty about the signals of players 2 and 3 (approximating $s_2 + s_3$ using M_2); player 2 has uncertainty only about s_3, which she approximates with M_1; and player 3 should be informed of the value of α and will guess the correct answer with certainty. Thus, the higher the player position in

predecessors $(I_3 = (s_{3,}g_1, g_2))$.

the sequence, the higher her chances to win a prize (note that $\Pr(s_2 + s_3 = M_2) < \Pr(s_3 = M_1)$).

Out of the PBE equilibrium path, any beliefs with the corresponding best reaction guesses would be acceptable. Anyway, in order to uniquely define an equilibrium we propose the following *belief criterion,* which seems the most reasonable:

$$
\begin{aligned}
\mathcal{M}_2(I_2) &= (\mu_2(I_2)(0, s_2, g_1), \mu_2(I_2)(1, s_2, g_1)) \\
&= ((1 - P_{21}(g_1)), P_{21}(g_1)) \\
\mathcal{M}_3(I_3) &= (\mu_3(I_3)(0, 0, s_3, g_1, g_2), \mu_3(I_3)(0, 1, s_3, g_1, g_2), \\
&\quad \mu_3(I_3)(1, 0, s_3, g_1, g_2), \mu_3(I_3)(1, 1, s_3, g_1, g_2)) \\
&= ((1 - P_{31}(g_1)) \cdot (1 - P_{32}(g_1, g_2)), (1 - P_{31}(g_1)) \cdot P_{32}(g_1, g_2), \\
&\quad P_{31}(g_1) \cdot (1 - P_{32}(g_1, g_2)), P_{31}(g_1) \cdot P_{32}(g_1, g_2)), \\
where &: \\
P_{i1}(g_1) &= \Pr_{i,1}(1|g_1) = \begin{cases} 0 & if \quad g_1 \leq M_2 \\ 1 & otherwise \end{cases}, \qquad i \in \{1, 2\} \\
P_{32}(g_1, g_2) &= \Pr_{3,2}(1|g_1, g_2) = \begin{cases} 0 & if \quad g_2 \leq M_1 + P_{21}(g_1) \\ 1 & otherwise \end{cases}
\end{aligned}
$$

In words, since the equilibrium prescribes that player 1 makes a guess $g_1 = s_1 + M_2$, when players 2 and 3 observe $g_1 \leq M_2$, they infer that player 1's signal is 0.

Similarly, player 3 knows that player 2 infers $s_1 = 0$ when $P_{21}(g_1) = 0$ and $s_1 = 1$ when $P_{21}(g_1) = 1$. Thus, since equilibrium prescribes that player 2 makes a guess $g_2 = P_{21}(g_1) + s_2 + M_1$, when player 3 observes $g_2 \leq M_1 + P_{21}(g_1)$, she infers that player 1's signal is 0.

Hence, with this beliefs system a PBE would be formed by the following players' strategies:

$$
\begin{aligned}
g_1(s_1) &= s_1 + M_2 \\
g_2(s_2, g_1) &= P_{21}(g_1) + s_2 + M_1 \\
g_3(s_3, g_1, g_2) &= P_{31}(g_1) + P_{32}(g_1, g_2) + s_3
\end{aligned}
$$

Finally, we note that in the experiment we use $p = \frac{3}{4}$, hence a PBE is obtained by substituting $M_1 = 1$ and $M_2 = 2$ in the expressions above.

3 The experimental design

In what follows, we describe the features of the experiments in detail.

Subjects. The experiment was conducted in 4 subsequent sessions in May, 2002. A total of 48 students (12 per session) were recruited among the undergraduate student population of the Universidad de Alicante.[4]

Treatment. The 4 experimental sessions were run in a computer lab.[5] In each session, subjects played 20 rounds of the game. In all 20 rounds of each session subjects played anonymously with the same opponents (that is, the group composition was kept constant throughout the sessions). Moreover, also player position was fixed. Instructions were provided by a self-paced, interactive computer program that introduced and described the experiment.

Payoffs. All subjects received 1000 ptas. (1 euro is approx. 166 ptas.) to show up. The fixed prize for each round was equal to 50 ptas.

Matching. As we said, subjects experienced one player position only in the guessing sequence for all treatments. We did so to simplify their decision making process (and data analysis). We also fixed group composition. We did so to enhance learning effects and to obtain a higher number of independent observations for each individual experimental session. After each round each agent was informed of his payoff and of the guess and the signal of each agent in his group in the current round, and additionally they could observe a table with the history of signals and guesses each agent in his group made in all previous rounds. This allows to enhance learning of agents.

Group size. As previously mentioned, all experimental sessions were characterized by a group size $n = 3$.

Random events. We have taken $p = 3/4$.

4 Results

Let us describe the results of the experimental sessions. We will divide the analysis into three parts. First we describe the outcomes, that is, the frequency with which players (we will refer to player positions as "player" here-

[4]Mainly, undergraduate students from the Economics Department with no (or very little) prior exposure to game theory.

[5]The experiment was programmed and conducted with the software z-Tree (Fischbacher [?]).

after) get the prize Then we turn to analyze behavior; to this aim we describe to which extent players behavior in the experiment adjust to the theoretical equilibria. Finally we analyze the correlation that exists between the guesses made by the players and the information sets they had.

4.1 Outcomes

The outcome results for the three treatments are resumed in *Table 1.* It shows the percentage over the total number of rounds played in which the players guess right (i.e. their guesses coincide with the sum of signals), hence it represents the frequency with which each player wins the prize. In brackets we display the theoretical predictions of the probability of guessing right (or, equivalently, winning the prize) when players follow the equilibrium strategy.

Player	Frequency of guessing right
1	40,51 (56)
2	50,32 (75)
3	61,08 (100)

Table 1

We can observe that the results reproduce qualitatively the theoretical predictions. The frequency of winning the prize is increasing in player position, as theory predicts. Nevertheless, the frequencies with which players win the prize is lower than the theoretical probabilities of guessing right. This suggests that players make errors and deviate from the equilibrium strategies in some rounds. We observe how the difference between the frequency of winning and the theoretical probability to win is increasing with player position (15,49 for player 1, 24,68 for player 2 and 38,92 for player 3). In sections 4 and 5 below we provide an econometric analysis of the deviations from equilibrium by players and relate it to accumulation of errors (error cascades).

4.2 Behavior

Now we turn to describe the behavior of the agents (again aggregating by player positions) to try to compare with the equilibrium behavior described in Section 2. In this section, we will restrict to describe the cases in which the path of actual play is consistent with the equilibrium.

In equilibrium, player 1 guesses 2, in case she has signal 0, or guesses 3, in case her signal is 1 (cf. Section 2), thus we only analyze player 2's behavior when she observes that player 1 has guessed either 2 or 3. Similarly, to analyze player 3's behavior we only consider paths of guesses of player 1 and 2 consistent with the equilibrium.

The information is presented in tables, referred to player positions. It is shown, for each information set, the frequency with which each possible guess is made with respect to the total number of cases in which the considered information set is reached. For each row (information set reached at equilibrium) the cell in bold represent the equilibrium best response of the player, as detailed in Section 2. Since we run 4 sessions of 20 rounds, and in each case we have 4 groups (of 3 players), we have a total of 960 observations (320 observations of guessing sequences).

Tables $2.1 - 2.3$ show the behavior of players $1 - 3$ respectively restricted to information sets that are compatible with the equilibrium. In each row it is presented the information set, which, for player 1, consists on her own signal (s_1); for player 2 consists on both her own signal (s_2) and the guess made by player 1 (g_1); and for player 3 consists both on her signal (s_3) and the guesses made by her predecessors (g_1 and g_2). In the table we can observe the percentage with which the player makes any of her possible guesses (0, 1, 2 and 3) over the total number of times the information set of the player is reached. The last column represents the frequency of equilibrium play aggregating for information sets.

Info. set: Signal 1	Guess 1				
	0	1	2	3	
0	0,93	26,85	**72,22**	0	59,18 %
1	0	9,62	37,98	**52,4**	

Table 2.1

We can observe that player 1 plays the equilibrium with a frequency of $59,18\%$. This frequency is increased when she gets signal 0 ($72,22\%$), meanwhile it is lower when her signal is 1 ($52,40\%$). The explanation relies on the fact that there may be agents that do not make use of their own information: given the priors ($p = 3/4$) the most likely answers are 2 and 3 (both with a probability $0,4219$), both of which are possible when player 1's signal is 1, this can explain the fact that in this case player 1 guesses 2 with a high frequency $(37,98\%)$; on the other hand, when player 1 has signal 0, she knows for sure that the answer is not 3 (which explain the frequency 0% for $g_1 = 3$), and thus she guesses 2 in a higher frequency $(72,22\%)$.

Info. Set pl2		Guess 2			% Eq. Play
Guess1	Signal2	1	2	3	
2	0	**39,22**	60,78	0	65,78 %
	1	7,55	**57,55**	34,91	
3	0	20,69	**75,86**	3,45	
	1	0	10	**90**	

Table 2.2

If we consider player 2 we observe that she plays the equilibrium in $65,78\%$ of the cases. The frequencies are higher in case player 1 guessed 3 (frequency $86,23\%$) from which player 2 should infer that player1's signal is 1, than when she guessed 2 (frequency $51,59\%$), from which she should infer $s_1 = 0$. An explanation for this fact could be that player 2 assigns positive probability to the event that player 1 is playing at random and thus she weights to some extent the priors when inferring player 1's signal.

Info. Set 3			Guess 3				Eq.Play (%)
G1	G2	S3	0	1	2	3	
2	1	0	**9,09**	72,73	18,18	0	63,23%
		1	0	**29,41**	58,82	11,76	
	2	0	0	**17,24**	82,76	0	
		1	0	0	**71,43**	28,57	
3	2	0	0	**37,5**	56,25	6,25	
		1	0	0	**64,29**	35,71	
	3	0	0	4,17	**87,5**	8,33	
		1	0	0	0	**100**	

Table 2.3

We can see that player 3 plays the equilibrium with a frequency of $63,23\%$. Again we observe that player 3 plays the equilibrium action much more frequently as

(i) Player 1 and player 2's guesses are higher.

(ii) Her own signal (s_3) is higher.

To highlight this fact, we note that: in the information sets in which player 1 guess 2 and player 2 guess 1, player 3 plays the equilibrium with a frequency of $21,43\%$ meanwhile in the information sets in which both player 1 and player 2 guess 3, player 3 plays the equilibrium in $95,89\%$, which is increased to 100% when she gets $s_3 = 1$.

This fact can be explained by the same argument used in player 2's case, that is, to consider that player 3 conceives that both player 1 and player 2 do not always follow the equilibrium strategy but play at random with some probability. In this case player 3 weights the priors ($p = 3/4$) when he tries to infers players 1 and 2's signals.

4.3 Correlation between guesses and information sets

Now we show the correlation among guesses and information sets by player position. We analyze the following correlations, which are resumed in Table 3.

(i) For player 1: correlation between her signal and her guess (s_1, g_1).

(ii) For player 2: correlation between the sum of her signal and the guess of player 1 and her guess $(s_2 + g_1, g_2)$.

(iii) For player 3: correlation between the sum of her signal and the guess of player 2 and her guess $(s_3 + g_2, g_3)$.

Player 1	0,49
Player 2	0,64
Player 3	0,75

Table 3

We find high positive correlations between information sets and guesses for the three players. This correlation is observed to be increasing with respect to the player position.

Hence we summarize this section by saying that the equilibrium of the game is a good predictor of our experimental data, but we observe that there is also the presence of noise. In the next two sections we rationalize these deviations from the equilibrium by taking into account learning considerations. Since we have fixed groups we are able to make panel data estimations and understand each agent play with regard to his own experience in the group.

5 Learning analysis

In the previous section we presented the results of the experiment aggregating the data for player positions. Nevertheless, we had fixed groups, and hence independent observations. Hence, we can do a deeper econometric analysis in order to understand agents behavior with respect to his own past experience. In our framework, it could be the case that some particular player is wrongly playing a strategy which does not correspond to his optimal best response. Hence, the fact that successors in the sequence do not take this into account would lead them to react suboptimally. We can conceive that players may have the ability to learn predecessors' strategies by means of past experience, i.e. the actual path of play of those agents throughout all previous periods. Thus even if a player is behaving according to a strategy which is different from the best response, succeeding players should be able to learn to play a best response to the actual strategy they observe.

5.1 Formal definitions

We say players are *optimal* when they react playing a best response with respect to their predecessors' strategies, which they infer throughout the experience from previous periods. We say players are *notionally optimal* when they play a best response with respect to their predecessors' theoretical best response. Hence it is clear that the case of full optimality, i.e. when all players play optimally, correspond to the situation of notional optimality.

We say there is *learning* when a player is able to react optimally to his past experience, i.e. to the strategy that he infers that his predecessors are playing. Nevertheless, learning may be imperfect. We say there is *imperfect learning* when a player's learning is less intense the higher the deviation from optimal behavior by his predecessors.

In order to analyze learning we recall we have a panel where there are 16 groups (16 subjects in each player position) and 20 rounds. We make logit estimations from period 11-20, so as to let players to use the first 10 periods to learn how other agents in the group behave. To analyze player 2, we first define a proxy for the behavioral strategy of player 1 he infers from his experience.

Assume player 1 is playing a particular behavioral strategy $\gamma = \{\gamma_1^{x,y}\}_{x\in\{0,1\},y\in\{0,1,2,3\}}$, not necessarily the equilibrium one, where $\gamma_1^{x,y}$ represents the probability with which player 1 makes a guess $g_1 = y$ when he has $s_1 = x$.

At period $t > 10$ plater 2 can construct a proxy for player 1's behavioral strategy $(\hat{\gamma})$ as follows: $\hat{\gamma}_1^{x,y}(t)$ would be obtained as the frequency with which player 1 choose $g_1 = y$ among all times he received $s_1 = x$ from period 1 to period $t-1$.

Hence, the posterior belief that player 2 has on player 1 having a signal $s_1 = 0$ given that at period t he has observed $g_1(t) = y$ is then calculated by applying Bayes' rule:

$$\Pr(s_1(t) = 0 \mid g_1(t) = y) = \frac{\frac{1}{4}\hat{\gamma}_1^{0,y}(t)}{\frac{1}{4}\hat{\gamma}_1^{0,y}(t) + \frac{3}{4}\hat{\gamma}_1^{1,y}(t)}$$

and $\Pr(s_1(t) = 1 \mid g_1(t) = y) = 1 - \Pr(s_1(t) = 0 \mid g_1(t) = y)$.

Then at any period $t > 10$ we can calculate the belief that any player 2 has that player 1 of his group (who has chosen $g_1(t) = y$) is playing optimally as:

$$b_1 = \Pr(s_1 = g_1^{*^{-1}}(y) \mid g_1(t) = y)$$

i.e. the variable b_1 represents the probability that player 2 assigns to player 1 to have played his best response given that he observes a guess $g_1 = y$.[6]

Now we can define the *optimal best response by player 2* to the behavioral strategy he has inferred from player 1 at period t, when he observes player 1's guess to be $g_1(t) = y$:

$g_2^*(s_2, g_1 = y)$ is set to be the action which maximizes the expected benefit of player 3, given that the beliefs about player 1's signal is represented by

[6] In all our calculations we consider any guess ($y \in \{0, 1, 2, 3\}$) by player 1, even if this guess is not compatible with the equilibrium. We consider the equilirium strategy we propose by aplying the criterium belief system proposed in section 2. This is done to have a higher number of observaions. We note that restricting to guesses compatible with the equilibrium does not change our results.

the variable b_1 and the belief about player 3's signal is represented by the priors $\left(\frac{3}{4}, \frac{1}{4}\right)$.

Finally at any round $t > 10$ and for any subject acting as player 2 we can define a binary variable which indicates whether he has played optimally with regard to player 1's behavioral strategy which he infers through his experience and the current guess he observes $g_1 = y$ as:

$$BR_2(t, g_1 = y) = \begin{cases} 1 & \text{if } \; g_2(t) = g_2^*(s_2, g_1 = y) \\ 0 & \text{otherwise} \end{cases}$$

Similarly, for player 3 we construct the variable b_2, which represents the probability that player 3 assigns to player 2 to have played optimally to his experience given the guesses he observes by players 1 and 2.[7] Then we also calculate $g_3^*(t, g_1, g_2)$ which is the action that maximizes the expected benefit of player 3, given that the beliefs about player 1 and player 2's signals are represented by the variables b_1 and b_2, respectively. Analogously we also can construct the binary variable $BR_3(t, g_1, g_2)$ which indicates whether a subject acting as player 3 has played optimally with regard to his predecessors' behavioral strategies that he infers through his experience and the guesses he observes in the current period.

5.2 The case of perfect learning

A situation perfect learning by an agent would be represented by the case where he is able to react optimally to the behavioral strategies of his predecessors inferred through experience.

For example, consider player 2. In the case of perfect learning would be represented by the fact that regardless the value of $b_1 \in [0, 1]$ he may observe he reacts optimally. This extreme case is represented in Figure 1.

[7] The calculations are similar to those for b_1 but more tedious and, hence, ommited.

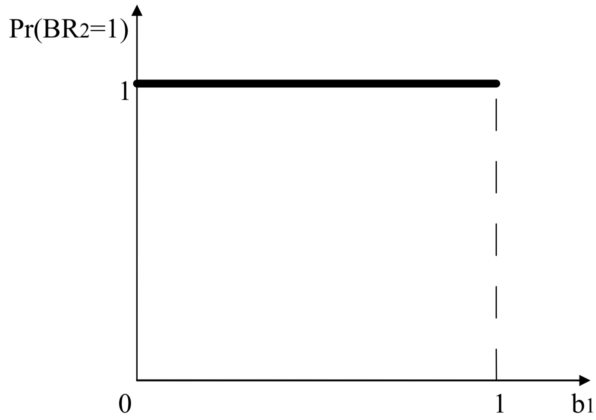

Figure 1

The case of perfect learning is represented when the probability of player 2 playing a best response $\Pr(BR_2 = 1)$ represented as a function of b_1 is an horizontal line at 1. One could also conceive that there are some errors but independently of b_1, in this case we would obtain an horizontal line at a lower level. For player 3, the case of perfect learning would be analogously represented by an horizontal line representing the fact that player 3 reacts optimally to the behavior of his predecessors regardless the observed values of b_1 and b_2.

5.3 Imperfect learning: econometric analysis

In this section we analyse the probability of optimal behavior of agents with regard to the behavior of predecessors. To this aim we make (panel data) logistic regressions.

For player 2 we regress the probability of behaving optimally with respect to b_1. The results are presented in Table 4 below:

BR2	Coef.	Std. Err.	z	P>|z|
b1	2.747572	.6980889	3.94	0.000
cons	-1.239713	.4699841	-2.64	0.008

Table 4: Random-effects logistic regression[8]

[8] Software used: STATA. Number of obs = 147. Group variable (i): group. Number of groups = 16. Random effects u_i ~Gaussian. Wald chi2(1) = 15.49. Log likelihood = -86.620567. Prob > chi2 = 0.0001.

The fact that the coefficient is significant and positive reflects the fact that there is *imperfect learning by player 2*: player 2's learning is less intense the higher the deviation from optimal behavior by player 1. Moreover if we represent graphically the predicted probability of player 2's best response with respect to b_1 we observe that the relation among both variables is very close to be linear. Figure 2 represents this relationship.

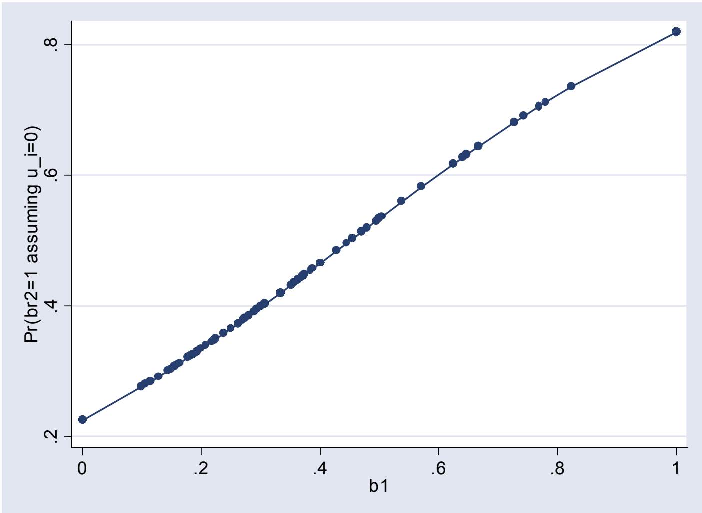

Figure 2

The explanation for the presence of imperfect learning is the following: when b_1 is high then optimal behavior by player 2 to his own experience coincides with notional optimality, hence this improves player 2's ability to react optimally and the probability of playing a best response is high. Differently, when b_1 is low (and hence the belief about player 1 reflects that he is not behaving optimally), then there is a conflict between the optimal behavior of player 2 to his own experience and notional optimality (optimal behavior with respect to the equilibrium of the game), hence player 2's ability to react optimally is lower when b_1 is small. This fact explain this positive and increasing relation between $\Pr(BR_2 = 1)$ and b_1.

Now we turn to analyze player 3 learning. We make a logistic regression of the probability of player 3 to play a best response as a function of b_1 and b_2. The results are shown in table 5 below.

BR3	Coef.	Std. Err.	z	P>|z|
b1	.7175597	.7322596	0.98	0.327
b2	1.307434	.6298127	2.08	0.038
cons	-.4995752	.4185546	-1.19	0.233

Table 5: Random-effects logistic regression[9]

We find that coefficient of b_2 is positive and significant while the coefficient of b_1 is also positive but not significant. We care of the fact that variables b_1 and b_2 are correlated. In fact if we run the above estimation without b_2 we get a significant coefficient of b_1. We have constructed a bivariate VAR model and found evidence that b_1 at period $t-1$ is a good predictor of b_2 at time t, as well as b_2 at time $t-1$ isn't a good predictor of b_1 at time t. This facts are consistent with the idea that b_1 affects player 3's probability to play best response through the effect on b_2. Hence we show in table 6 below the logit estimation where we only use b_2 as explanatory variable.

BR3	Coef.	Std. Err.	z	P>|z|
b2	1.637885	.5426834	3.02	0.003
cons	-.3373284	.3831164	-0.88	0.379

Table 6: Random-effects logistic regression[10]

The fact that the coefficient of b_2 is significant and positive reflects the fact that there is *imperfect learning by player 3*: player 3's learning is less intense the higher the deviation from optimal behavior by his predecessors. Moreover, if we represent graphically the predicted probability of player 3's best response with respect to b_2 we observe that the relation among both variables is also very close to be linear. Figure 3 represents this relationship.

[9]Software used: STATA. Number of obs = 120. Group variable (i): group. Number of groups = 16. Random effects u_i ~Gaussian. Wald chi2(1) = 9.79. Log likelihood = -71.121655. Prob > chi2 = 0.0075.

[10]Software used: STATA. Number of obs = 120. Group variable (i): group. Number of groups = 16. Random effects u_i ~Gaussian. Wald chi2(1) = 9.11. Log likelihood = -71.60311. Prob > chi2 = 0.0025.

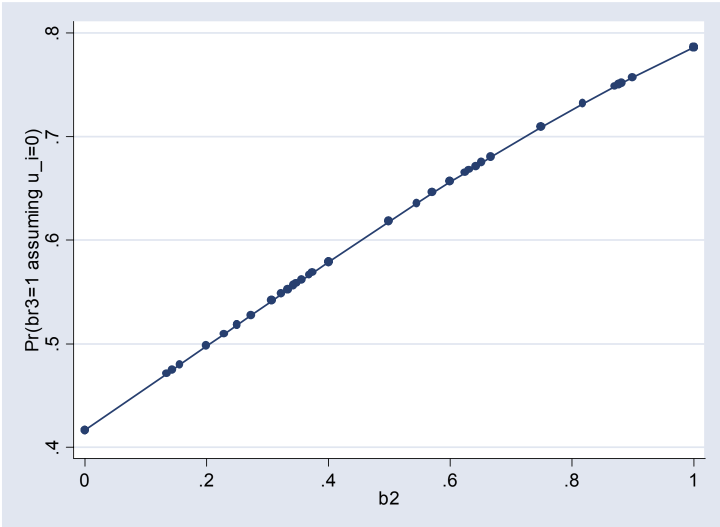

Figure 3

The explanation for the presence of imperfect learning by player 3 is analogous to the argument we developed above for player 2: the lower b_2, the higher the conflict among optimal behavior to the experience and notional optimality. Hence there is a positive and increasing relation between $\Pr(BR_3 = 1)$ and b_2.

6 Error cascades

The presence of imperfect learning generates an error cascade. An *error cascade* is the situation where the deviation from learning by a player increases with respect to the deviations from learning by preceding players.

In our experiment we have found the presence of imperfect learning by players 2 and 3. Hence, the higher the deviation from optimal behavior by player 1, the higher the error made by player 2. Moreover, the higher the deviation from optimal behavior by player 2, the higher the error by player 3. Hence the presence of error cascades represents an accumulation of errors through the guessing sequence.

6.1 A theory of error cascades

In this section we present a simple model, which may be helpful to understand the error cascade phenomenon.

6.1.1 The main idea

There is a set $N = \{1, 2, ..., n\}$ of players who make guesses according to a pre-specified sequence: agent 1 is the first and for any $i \in \{2, 3, ..., n\}$ agent i guess after agent $i - 1$.

Suppose that the probability to guess with a best response for a player, say $i \in N$, depends on the belief that player $i-1$ is guessing with a best response and assume that this dependence is described by the following function:

$$\Pr(BR_{i,t}) = \alpha_0 + \alpha_1 b_{i-1,t} + \varepsilon_{i,t} \quad (1)$$

where $b_{i-1,t}$ denotes the belief of player i at time t (based on past experience) on the probability that player $i - 1$ guesses using a best response and $\varepsilon_{i,t}$ is an error term for player i at time t.

Note that in the long run $\Pr(BR_{i,t}) = b_{i,t}$ and we can write equation (1) as:

$$b_{i,t} = \alpha_0 + \alpha_1 b_{i-1,t} + \varepsilon_{i,t} \quad (2)$$

If coefficients in equation (2) satisfy the following conditions:

$$\alpha_1 < 1; \quad \alpha_0 + \alpha_1 < 1 \quad (3)$$

the value of $b_{i,t}$ converges to a value $\hat{b} < 1$ as i goes to infinity, independently from initial conditions. In this way we can identify an error cascade when conditions (3) hold. See figure 4 for a graphical intuition

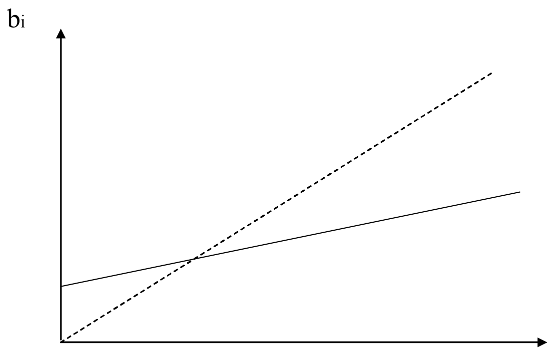

Figure 4

We note that . If if there are many agents in the guessing sequence we can assume that the slope in equation (2) is near to zero, therefore the value of b converges to the value of the intercept.

6.1.2 A simple model of error cascades

Assume that, in the long run, the probability to guess with a best response follows the rule:

$$\begin{aligned} b_i &= 1-\varepsilon_i-\rho_{i-1}(1-b_{i-1}) \quad \forall i>1 \\ b_1 &= 1-\varepsilon_1 \end{aligned} \tag{4}$$

where $\varepsilon_i \in [0,1]$ $\forall i \in N$ represents the probability of an independent (not induced) mistake. Therefore, for player i, the probability to guess with a best response depends on the value of and on the probability of mistake of player $i-1$. This effect is captured by the last term of equation (4) and depends on the value of the coefficient ρ_{i-1}.

Substituting recursively the equations in (4) we obtain the following equation for player i:

$$b_i = 1-\varepsilon_i-\rho_{i-1}\varepsilon_{i-1}-\prod_{j=i-1}^{i-2}\rho_j\varepsilon_{i-2}-\prod_{j=i-1}^{i-3}\rho_j\varepsilon_{i-3}-\ldots-\prod_{j=i-1}^{1}\rho_j\varepsilon_1$$

that in more compact form is:

$$b_i = 1-\sum_{j=1}^{i}\left(\varepsilon\prod_{j}^{i-1}\rho_j\right) \tag{5}$$

To simplify assume that:

$$\begin{aligned} \rho_1 &= \rho_2=\ldots=\rho_n=\rho \\ \varepsilon_1 &= \varepsilon_2=\ldots=\varepsilon_n=\varepsilon \end{aligned}$$

Therefore equation (5) becomes:

$$b_i = 1-\varepsilon\frac{1-\rho^i}{1-\rho} \tag{6}$$

We can identify two polar cases:

1. *Full error cascade.* We note that $\lim_{\rho \to 1} b_i = 1 - i \cdot \varepsilon$. That is, there is a (full) accumulation of errors in previous positions.

2. *No error cascade.* We note that $\lim_{\rho \to 0} b_i = 1 - \varepsilon$. The errors of previous players don't affect the probability to guess using a best response.

In the intermediate cases we find that when $\rho \in (0, 1)$:

$$\lim_{i \to \infty} b_i = 1 - \frac{\varepsilon}{1 - \rho} = \hat{b}$$

That is, the probability to guess with a best response converges at as the position in the sequence rises.

The condition $\rho < 1 - \varepsilon$ assures $\hat{b} > 0$, i.e. in a infinite guessing sequence, the probability to play a best response is strictly positive.

Given a value of $i \in N$ the condition for $b_i > 0$ is $\varepsilon \frac{1-\rho^i}{1-\rho} < 1$, i.e. for a given value of ε there exists a value $\hat{\rho}$ such that for all $\rho < \hat{\rho}$ the condition is verified.

6.2 Data evidence

We have a game with 3 players ($n = 3$). Hence, even if asymptotic results do not make sense, we are able to find significant evidence of error cascades, which are summarised below.

In the data we observe that if player 1 guesses with best response then player 2 also guesses with best responses with high probability. In fact, the higher the posterior beliefs player 2 has on player 1 playing a best response (which are calculated on behalf of his experience over previous periods), the higher the estimated probability with which player 2 plays a best response is. Hence if eventually player 1 learns to play the best response then player 2 will also learn to react optimally. For evidence of this fact recall table 4 and figure 2 above. This effect also holds in the relation between players 2 and 3: player 2 playing a best response makes player 3 to play a best response with high probability. In fact, the higher the posterior beliefs player 3 has on player 2 playing a best response (which are calculated on behalf of his experience over previous periods), the higher the estimated probability with which player 3 plays a best response is. For evidence of this fact recall table 6 and figure 3 above. We find that, in many observations, there are significant deviations from the best response by player 1, which results in subsequent deviations by players 2 and 3. This evidence represents the presence of error cascades, i.e. the fact that players in former positions learn (do not learn) to play the best response has a positive (negative) effect on later players to play their best response. This is summarized by three facts: first, when player 1 plays a best response it is very likely to find notional optimality in the whole group; second, there is imperfect learning, since agents in positions 2 and 3 adapt by best response (to their experience) only imperfectly: the less perfect the more preceding players deviate from optimality, and third, imperfect learning generates the presence of error cascades.

Now we wonder which matters affect the ability of player 1 to play the best response, since he does not have any predecessor in the sequence. In table 7 we present the mean frequency (over all rounds) of optimal behavior for each player position.

	# obs.	Mean
BR_1	316	0.5917
BR_2	222	0.6216
BR_3	153	0.6862

Table 7

What is the reason why player 1 does not eventually learn to play the best response? Why do we observe a higher frequency of best response play in higher player positions? This is explained by the fact that deviating from the best response is less costly the lower the player position is. Note that the lower the player position, the higher the deviations from optimal behavior. In fact we find significant deviations from the best response by player 1. We explain this fact by the cost each player has of deviating from the best response. Note that, in this case, in which we have only three players, there is a significant difference of cost among them. This cost is higher the lower the player position.

To illustrate this idea, we calculate the cost from deviating for the best response for each player position in the equilibrium path. First we determine which is the expected cost player 1 would have from deviating from the best response. In table 8 we show the expected profit of player 1 by making a guess resulting to add to his own signal 0, 1 and 2.

Player 1's guess	Expected profit
$s_1 + 2$	9/16
$s_1 + 1$	6/16
s_1	1/16

Table 8

Hence, the cost of deviating from the best response to the second better alternative is $\frac{3}{16}$.

Now, let us assume that player 1 play the best response in order to calculate player 2's cost of deviating in the equilibrium path. In table 9 we present the expected profits of those guesses with which player 2 has positive probability of winning.

Player 1's guess	Expected profit
$s_1 + s_2 + 1$	3/4
$s_1 + s_2$	1/4

Table 9

Hence, the cost of deviating for player 2 is $\frac{1}{2}$.

Now, let us assume that player 1 and 2 play the best response in order to calculate player 3's cost of deviating in the equilibrium path. For player 3, the profit to make a guess equal to $s_1 + s_2 + s_3$ is equal to 1, and any other guess provides him with zero payoff. Hence, the cost of deviation for player 3 is 1. In table 10 we summarise the cost each player has to deviate from the equilibrium path. We observe how this cost is increasing with respect to the player position

Player 1's cost	3/16
Player 2's cost	1/2
Player 3's cost	1

Table 10

Now we turn to analyse the cost agents had to deviate from optimal behavior (to his experience) in our experimental sessions. Table 11 presents the average cost each agent position had to deviate from optimal behavior.

Player 1's average cost	0.1875
Player 1's average cost	0.3003
Player 1's average cost	0.4853

Table 11

We observe that this empirical cost is also increasing in player position. So both theoretically, in the equilibrium path, and empirically from our data we obtain the same result: the cost agents face to deviate from optimal behavior is increasing in the player position.

To summarize, all the considerations above imply that deviations from notional optimality of the whole group must be associated to initial deviations by the first player, since we observed that optimal behavior by player 1 resulted in learning to play optimally by subsequent players. Differently we observed the error cascade phenomenon: when player 1 deviates from optimal behavior results in errors by successors, which accumulate We have conjectured which facts would lead player 1 to not eventually learn, and hence to explain deviations from notional optimality. We show that a possible explanation is the cost players face from deviating from the best response. This cost is observed to be increasing with respect to the player position in the sequence. Thus, the first player is the one who has the lowest cost of deviating from the best response, which makes him to have the lowest incentives to follow the optimal strategy.

7 Related Literature

Anderson and Holt (1997), henceforth referred to as AH, develop an experiment based on the theoretical framework proposed by Bikhchandani et *al.* (1992). In their case there are two possible states of the world. At each round agents are randomly ordered, and one state of the world is randomly selected with equal probability. Then, each agent receives a private signal about the true state of the world and they have to make sequential public binary decisions (according to the realized ordering). Whenever an agent guesses the right state of the world she gets a fixed prize and she gets zero otherwise, in this sense the payoff structure is identical to ours. But, differently to us, in their setup, the signal an agent receives is informative in that the probability that the signal will match the true state of the world is higher than 1/2, i.e. they present a binary-signal-binary-action model.[11] Hence, they analyze the presence of cascade behavior: agents' choices of guesses which disregard their own private information and instead follow predecessors' decisions. This behavior is individually rational since, in their model, the set of states of the world coincide with the set of signals an agent can receive. Differently, in

[11] Differently in our case the signal agent receive is informative with probability one.

our model, the set of states of the world consists on vectors where each component is the signal an agent may receive, and hence a player's signal is always informative about the true state of the world regardless predecessors' guesses. Hence, in our framework an information cascades never constitutes an equilibrium. AH observe a high frequency of optimal cascade behavior, but which nevertheless is lower than theoretical predictions. Hence they include an econometric analysis of errors: they assume agents allow for the possibility of errors in earlier decisions when making their choices. These errors are (recursively) estimated assuming a logistic distribution of independent shocks to expected payoffs. This analysis allows to explain many of the observed patterns of behavior.

There has been a branch of experimental papers which have been framed in the setup of AH to analyze various interesting aspects of the problem at hand, which we now summarize.

Kübler and Weizsäcker (2003) examine the robustness of information cascades in laboratory experiments by considering the possibility of introducing costly signals: apart from replicating AH's situation where each player obtain a signal for free, they study the case of costly signals where players decide whether or not to obtain private information at a small but positive cost. In the equilibrium of this game only the first player buys a signal and the remaining agents herd on the decision of such agent. Nevertheless the experimental results show that the equilibrium prediction performs poorly since too many signals are bought. They explain these observation by allowing for different error rates on different levels of reasoning.[12] They find that the subjects' inferences become significantly more noisy on higher levels of the thought process, and that only short chains of reasoning are applied by the subjects.

Kraemer et al. (2000) also analyse the case of costly signals. They focus on the case were acquisition of signals are publicly observed. Their data also reveal that participants bought on average too many signals as compared to a bayesian rational individual.

Hung and Plot (2001) study different rewarding schemes by analysing three different types of organization: (1) they replicate AH's setup where agents were rewarded according to whether their announced decision was right or not; (2) agents were rewarded according to whether a majority of

[12] The depth of the subjects' reasoning process is estimated using a statistical error-rate model.

announced decisions were right or not, and under this institution, they observe that the instance of information cascades is sharply reduced; and (3) agents are rewarded more according to whether their personal announced decision was the same as the majority decision than they were rewarded if their decision was correct, and, under this institution, they observe substantial information cascades.

Anderson (2001) expands on the error analysis presented in AH by examining error rates under different payoff conditions; the results indicate that rewarding correct decisions reduces the amount of decision error as compared to the no rewarding case; however, increasing the payoff for a correct decision does not reduce the errors over the range of payoffs considered.

Willinger and Ziegelmeyer (1998) test the possibility for more informed agents to shatter potential information cascades: they replicate AH's experiment so as to compare to a treatment were publicly announced additional signals are introduced within the decision sequence; they observe that the "shattering mechanism" reduces the number of observed cascades and therefore tends to improve decisions.

Oberhammer and Stiehler (2003) implement the Becker-DeGroot-Marschak (1964)'s mechanism in a cascade experiment, i.e. they ask subjects to submit maximum prices for participating in the prediction game, and the stated price limits are used as indicators of their probability beliefs. This allows them to study in more detail than AH the individual updating behavior. They find that the inclusion of errors does not significantly improve the explanatory power of the standard approach.

Koessler and Ziegelmeyer (2000) reinterpret AH's data by means of defining a different tie-breaking rule and considering the possible subjects' beliefs on others' tie-breaking rules.

Huck and Oechssler (2000) analyze whether observed cascades in the lab are due to Bayesian updating. They develop an experiment where they control for subject's beliefs by confronting them with a hypothetical sequence of decision taken by hypothetical predecessors, where subjects were explicitly told that all previous decisions were taken rationally. They find that the simple heuristic "follow your own signal" does much better in explaining the data than bayesian rationality. They argue that AH's decisions tasks were much simpler than theirs, which explains why it seems AH's subjects seemed to apply Bayes rule quite well. Nevertheless they argue that a closer inspection of the more difficult decisions in AH reveal that the frequency of bayesian play is quite lower.

Nöth and Weber (2003) extend AH's setup by introducing two instead of one signal quality, in order to be able to distinguish clearly between "counting heuristic" behavior and bayesian updating. In their case potential cascades can collapse if an agent receives high quality information or if somebody believes more in her private information (overconfidence) than justified by Bayes' rule. They observe that subjects do not make their predictions using Bayes' rule but they employ identifiable heuristics, which put too much weight on private information.

Kremer and Nöth (2000) design an experiment similar to Nöth and Weber (2003) where they extract subjects' probability judgments before they reveal their decision. They observe that an "Anchoring and Adjustment" heuristic, in which the own private information provides the anchor and the adjustment is based on the available public information, explains the data better than bayesian updating.

Çelen and Kariv (2003a)-(2003b) are the papers which are closest to our framework. They analyse a situation where each agent receives a signal from the continuous space $[-10, 10]$ with uniform probability, and players have to guess sequentially whether the sum over the signals of all players is "positive" or "negative".

Çelen and Kariv (2003a) focus their analysis to differentiate information cascades from herd behavior in the lab. They rely on the differences between information cascades and herd behavior stressed by Smith and SØrensen (2000), who define an *informational cascade* as the situation where an (infinite) sequence of agents ignore their private information when making a decision, and *herd behavior* as the situation where an (infinite) sequence of agents make an identical decision, not necessarily ignoring their private information. To be able to make this distinction in the lab they use the strategy method in order to elicit subjects beliefs, i.e. instead of choosing an action directly, before an agent is informed about the realization of his signal they ask him to set a cutoff such that, if the signal is higher than the cutoff, the action "positive" is chosen and if the signal is lower than the cutoff, the action "negative" is selected. Hence, in their model, an information cascade never constitutes an equilibrium, but herd behavior is expected, i.e. situations where individuals become more and more likely to imitate but their actions may still provide information. They observe that in the lab both herd behavior and cascades occur frequently. Then they explain the arise of cascades as a particular deviation from Bayes rationality. This is done by allowing the possibility that subjects make errors and that they incorporate the possibility that others are making errors into their beliefs.

Çelen and Kariv (2003b) is based on the theory of Çelen and Kariv (2003c) and they explore the difference between social learning under perfect and imperfect information. In the former case each agent is able to observe the guesses of all preceding players, while, in the later, each agent only observes his immediate predecessor's guess. They find that imitation is much less frequent when players have imperfect information, even less frequent than the theory predicts.

Finally it is worth mentioning Allsopp and Hey (1999), who carry out experiments to test the model of Banerjee (1992). They consider the case which includes the possibility that people do not receive a signal, and in addition there is an infinite set of possible decisions. they find that herding occurs less frequently than predicted by Banerjee's framework.

To conclude this section we make a comparison among the econometric techniques used in some of these papers to analyse agents' deviation from the equilibrium prediction and our analysis of errors. We will focus in the papers by Anderson and Holt (1997) and Çelen and Kariv (2003a-2003b).

Differently to our framework, in the experiment of Anderson and Holt individuals face up a binary decision. Their econometric setup uses a probability matching, i.e. the probability to use a given action is an increasing function of the expected payoff. Specifically they use an exponential function. Therefore the errors have an effect on subsequent players through the effect that a noisy signal has on the expected payoff. This results in a recursive error structure, i.e. it is necessary to estimate the level of errors of a player in order to compute the expected payoff of subsequent players. The difference with our model is that we only use one variable capturing the informative signal and the level of mistake. This affects directly the probability to play the best response without any consideration on the expected payoff. Similarly, more errors by preceding players result in higher rates of deviations from best response by subsequent players.

The econometric treatment in Çelen and Kariv differs to Anderson and Holt's and ours in that they face up to a continue variable (as dependent one). In their model individuals are rational or irrational with a given probability. The rational individual reacts in a bayesian way to predecessors' errors. Then his decision can be rational or irrational with a given probability. This resulting in a recursive model of errors. But differently that in our model the deviations of previous players do not affect the deviations of subsequent players. It only affects to the decision of the rational player but it does not to the frequency of deviations.

Finally it is worth mentioning that the fact of having fixed groups and fixed player positions allows us to have independent observations and hence to use panel data econometric techniques, which is not possible in the related papers in the literature

8 Conclusion

We have proposed a sequential guessing game with incomplete information with no strategic conflict among agents. The equilibrium strategy for agents consists into decode predecessors' signals from their guesses and to take expectation for successors' signals from the priors. Our experimental data show that the equilibrium outcome is a good predictor for subjects behavior, since the frequencies of actual play reproduce qualitatively the equilibrium predictions. Nevertheless deviations from equilibrium are observed, and hence there is a presence of subjects' errors. The fact of having fixed groups allows us to make a panel data econometric analysis where we study agents behavior with respect to their own experience of play.

We make logit estimations where we regress the players' probabilities to play optimally with respect to the belief, inferred from their experience, that predecessors are playing optimally. We consider there is learning when players are able to react optimally to their past experience. We find the presence of imperfect learning, since the level of learning of a player is less intense the higher the deviation from optimal behavior by his predecessors. The intuition behind the presence of imperfect learning is that when preceding players deviate from optimality, then for a player to play optimally with respect to his own experience becomes contradictory to play a best response to the equilibrium of the game. The higher this conflict is, i.e. the higher the deviation by optimal behavior by predecessors, the lower the agent is able to react optimally. We argue that imperfect learning results in error cascades, the situation where errors by agents in the sequence produce errors in following players, hence errors are accumulated through the guessing sequence. Hence deviations from notional optimality of the whole group may be mainly explained by deviations by the first player and the subsequent accumulation of errors.

Finally we explain what may lead the first player to deviate from his equilibrium strategy. In fact we observe that the first player is the one who deviates from optimal behavior with a higher frequency, and the higher the player position the higher the frequency of optimal behavior. This can be explained relying on the cost agents face to deviate from optimal behavior. We show that in equilibrium the cost to deviate from the best response in terms of expected payoffs is increasing with the player position, hence the first player in the sequence is the one for whom it is less costly to deviate. Empirically, from our data we also find that the average cost of deviating from optimal behavior is also increasing with respect to player position. Hence, we claim that the first player is the one with less incentives to behave optimally among all players, and thus this is the player who presents a higher frequency of deviations. Nevertheless we find that even if, in average, successors played optimally with a higher frequency, we remark the fact that the frequency of these best responses is lower the higher the deviations from optimality by the first player due to the error cascade phenomenon.

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