Competition and Privatization

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Abstract

In this paper we study the interaction between privatization and competition (liberalization) in the market. Privatization is understood as a change in the objective of the owners of the firm. A public firm maximizes social welfare and a private firm maximizes profits. We find that this difference in objectives matters less the greater the level of competition because then the behavior of a welfare maximizing firm converges to the behavior of a profit maximizing firm. This result implies that privatization works better the greater the competition. On the other hand, the revenues obtained from privatization are decreasing with the level of competition. This implies that privatization aimed to raise revenues impose a cost in terms of allocative efficiency.

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1. Introduction

The recent privatization experience in some Western European countries suggests that the gains of privatization have fallen short of what initially expected. The U.K. privatization history shows that the cost reductions in the former public firms have not brought the expected price reductions. Thus, the gain in productive efficiency has not touched the consumer (see Newbery and Pollitt 1997). Green and Newbery (1992) analyze the British electricity spot market and comment that the whole process was handicapped since little competition was introduced at the start. They argue that the process would have worked better if the industry had been divided into five firms instead of the actual two operators.

This evidence shows that other considerations than efficiency in the industry came into play in the privatization process. In particular, the government budget constraints seems to have had great influence in the privatization decisions. Waverman and Sirel (1997) survey the privatization experiences in European telecommunications market and argue in the German case:

> "A key to understanding German restructuring in this area—and a potential roadblock to true competition—is German's desire to utilize proceeds from Deutsche Telekom privatization towards reducing government debt to meet the Maastrich criteria for monetary union. Greater competition in telecommunications would presumably reduce the future expected profits of DT, and thus cause the privatization to raise less revenue."

In this paper we study the interaction between privatization and competition (liberalization) in the market. The level of competition is measured in two different ways: as the difference in the unit cost of production between the two firms and the degree of product differentiation. Privatization is understood as a change in the objective of the owners of the firm. A public firm maximizes social welfare and a private firm maximizes profits. Our first result is that this difference in objectives matters less the greater the level of competition since the behavior of a welfare maximizing firm converges to the behavior of a profit maximizing firm. This result has a very nice implication on the effects of privatization: it works better the greater the competition. On the other hand, the revenues obtained from privatization are decreasing with the level of competition. This implies that privatizations aimed to raise revenues impose a cost in terms of allocative efficiency.

In this paper we build a model in which there are two firms, a public and a private firm. The private firm is a profit maximizer. It is assumed that the government cannot set directly prices. We assume that the government gives a cost subsidy to the public firm per unit produced and the firm is instructed to maximize profits after the subsidy. The subsidy is set to maximize social welfare. We show that the optimal subsidy is decreasing with the level of competition. This result is used to analyze the effects of privatization. The welfare loss of privatization is increasing with the level of the optimal subsidy and, therefore, decreasing with the level of competition in the market. These results are robust to an extension where we allow the private competitor to strategically manipulate the incentives given to his manager.

Our result that privatization always reduces welfare should be interpreted very carefully since we are abstracting away from productive efficiency considerations. In practice, the overall assessment of privatization requires to compare the losses in allocative efficiency to the gains in productive efficiency. Our model tells us that the negative part of privatization is decreasing with the level of competition.

So far, we have assumed that firms do not collude after privatization. If this were not the case, the effect of collusion would have to be taken into account in the assessment of privatization. We add a simple extension to our model, following Barcena-Ruiz and Espinosa (1996) and we obtain that a public firm and a private firm never collude, whereas two private firms will do it.

Our paper is related to others that also consider privatization a change in the objective of the firm, although they do not consider that ownership is separated from management. De Fraja and Delbono (1989) obtain in a Cournot setting that privatization may increase welfare if competition, measured as the number of firms in the market, is high enough. Anderson and De Palma (1998) show in a Bertrand setting that privatization always reduces welfare given the number of competitors. However, it may have a positive effect since higher profits in the market will attract new competitors.

In Section 2 we analyze the case in which the public firm receives a subsidy and the private firm maximizes profits. In Section 3 we study the case in which both firms strategically manipulate their managers' incentives. In Section 4 we introduce the possibility of firms making use of a collusive device. Section 5 concludes.

2. The Basic Model

2.1. Public Monopoly

Let us assume that the production of a good is controlled by a public monopoly. Unit cost of production is denoted by c. The demand for the good produced by the monopoly is given by

$$q = A - p, \tag{2.1}$$

$$A > c,$$

This demand is derived from the maximization problem of a representative consumer endowed with a utility function separable in money (denoted by m) given by:

$$V(q) = Aq - \frac{1}{2}q^2 + m \tag{2.2}$$

Social welfare maximization implies that the price should be set equal to the marginal cost of production. One standard way to achieve the optimal allocation is to subsidize the production cost of the firm and to instruct the manager to maximize the profit after the subsidy,

$$pq - cq + (1 - \alpha)cq, \tag{2.3}$$

where $(1-\alpha)$ is the fraction of the total cost that is subsidized. Thus, the manager sets the price equal to

$$p^m = \frac{A + \alpha c}{2}.$$

To achieve the optimal allocation, the government chooses the subsidy so that the monopoly price equals the marginal cost of production. This is obtained when

$$\alpha^m = 2 - \frac{A}{c},$$

which implies that the subsidy is positive.

2.2. Mixed Duopoly

Let us assume that the government opens the market to entry of private firms. For the sake of simplicity let us think of the case in which only one new firm enters the market. Firms set prices. The level of competition that this new firm brings into the market depends crucially on two different points: the cost level of the new firm and the degree of product differentiation.

To distinguish both firms we will use the subindex 0 for the public firm and the subindex 1 for the private firm. The public firm produces good 0 and the private firm produces good 1. Their demands are given by:

$$q_j = \frac{A}{1-h} - \frac{1}{1-h^2} \cdot p_j - \frac{h}{1-h^2} \cdot p_{1-j}, \ j = 0, 1 \tag{2.4}$$

$$A > 0, \text{ and } -1 \leq h \leq 0. \tag{2.5}$$

Those demands are derived from the maximization process of a representative consumer, (see Singh and Vives 1984) endowed with a utility function separable in money (denoted by m) given by:

$$V(q_0, q_1, m) = U(q_0, q_1) + m,$$

$$U(q_0, q_1) = A(q_0 + q_1) - \frac{1}{2}\left(q_0^2 - 2hq_0q_1 + q_1^2\right). \tag{2.6}$$

Observe that this utility function when the private firm does not produce coincides with the one shown in expression (2.2). The parameter h represents the degree of product differentiation. Unit costs of production are denoted by c_0 and c_1, respectively. We assume that the public firm is more efficient than the entrant since it is able to exploit its incumbent position. Nevertheless, we impose an upper bound on the private firm cost,

$$c_1 < \frac{A + c_0}{2},$$

so that entry affects competition. The private firm is assumed to maximize profits and the public firm maximizes after subsidy profits (expression shown in equation (2.3)). The equilibrium allocation and prices are

$$q_0 = \frac{A}{(2+h)(1-h)} - \frac{2-h^2}{(1-h^2)(4-h^2)}\alpha c_0 - \frac{h}{(1-h^2)(4-h^2)}c_1, \tag{2.7}$$

$$p_0 = \frac{A(1+h)}{2+h} + \frac{2}{4-h^2}\alpha c_0 - \frac{h}{4-h^2}c_1, \tag{2.8}$$

$$q_1 = \frac{A}{(2+h)(1-h)} - \frac{2-h^2}{(1-h^2)(4-h^2)}c_1 - \frac{h}{(1-h^2)(4-h^2)}\alpha c_0, \tag{2.9}$$

$$p_1 = \frac{A(1+h)}{2+h} + \frac{2}{4-h^2}c_1 - \frac{h}{4-h^2}\alpha c_0. \tag{2.10}$$

The government chooses α to maximize social surplus. The optimal subsidy satisfies

$$\frac{dW}{d\alpha} = \frac{\partial W}{\partial q_0}\frac{\partial q_0}{\partial \alpha} + \frac{\partial W}{\partial q_1}\frac{\partial q_1}{\partial \alpha} = 0. \tag{2.11}$$

Our first result, shown in Proposition 1 below, is that after liberalization the optimal subsidy does not need to be as high as that in the monopoly case. The reason for this is twofold: In one hand, competition itself brings profit margins down. On the other hand it is efficient to have a positive margin. This last point can be seen after rewriting expression (2.11),

$$\frac{dW}{d\alpha} = (p_0 - c_0)\frac{\partial q_0}{\partial \alpha} + (p_1 - c_1)\frac{\partial q_1}{\partial \alpha} = 0. \tag{2.12}$$

Since the price margin of the private firm will be positive and an increase in α (a decrease in the subsidy) increases output of the private firm and decreases production of the public firm, it follows that the margin of the public firm will be positive.

Proposition 1. *After liberalization, the optimal subsidy decreases.*

This proposition tells us that competition can substitute regulation. To analyze the relationship between competition and regulation, measured as the level of the optimal subsidy, we need to provide a measure for the degree of competition. There are two natural ways of measuring the level of competition in our context: first, for a given degree of product differentiation, the level of competition is decreasing with the difference in costs. Secondly, given symmetric costs, the level of competition is decreasing with the degree of product differentiation.

Proposition 2. *The optimal subsidy is increasing with the marginal cost of the private firm and, with symmetric costs, it is also increasing with the degree of differentiation.*

The focus on the level of subsidies is important because it will be useful to determine the welfare loss of privatization. In our framework, privatization simply implies that the government gives away the right to affect the level of competition through subsidizing the public firm. Therefore, intuitively, the greater the optimal subsidy, the greater the welfare loss. Observe that this can be clearly seen in the expression of the welfare loss below, that shows that it is proportional to the optimal subsidy, $1 - \alpha^*$.

$$\bigtriangledown W = (1 - \alpha^*)^2 c_0^2 \cdot \left(\frac{4 - 3h^2}{32 - 48h^2 + 18h^4 - 2h^6} \right). \tag{2.13}$$

The next proposition should be understood under the light of the previous proposition.

Proposition 3. *The welfare loss of privatization is increasing with the marginal cost of the private firm and, with symmetric costs, it is also increasing with the degree of differentiation.*

Remark 1. *The profits of the privatized firm are increasing with the marginal cost of the competitor and, with symmetric costs, are also increasing with the degree of differentiation.*

The welfare loss brought by privatization is lower the higher the level of competition. However, the revenues obtained selling the public firm will be lower in those cases. Therefore, raising government revenues by means of privatization is incompatible with optimal resource allocation.

3. Strategic Managerial Incentives in a Mixed Duopoly

Observe that profits after the subsidy, shown in expression (2.3), can be reinterpreted as the weighted sum of profits and sales,

$$\alpha (pq - cq) + (1 - \alpha) pq.$$

In this case $1-\alpha$ can be interpreted as a measure of how far is the public firm's objective from profit maximization. If α were equal to one (the no subsidy case), the firm would maximize profits. A reduction in α diverts the firm further away from profit maximization. Then, the results in the previous section can be reinterpreted in the following way: as competition becomes more intense, public firms become more profit oriented; in other words, the objective function resembles more closely that of the private firm. A similar result is obtained with Cournot competition by Matsumura (1998). He finds that the objective of the public firm becomes more profit oriented when competition is allowed.

As the effect of privatization is to change the objective function of the firm, this change will be less severe the more intense the level of competition in the market; therefore, welfare losses will be lower.

This type of incentives has been previously used in the literature. See, for instance, Fershtman and Judd (1987). In this context, we can think of the private firm in the previous section as one that did not exploit the strategic dimension of incentives. We now turn to analyze the case in which both firms use this instrument. Let us call α_0 the incentive for the manager of the public firm and α_1 the incentive for the manager of the private firm. Unlike Fershtman and Judd (1987) and Barros (1995), we assume that owners of firms have perfect information about market demand and cost functions. The only purpose of delegation is to make credible a behavior that would not be credible otherwise. As in Fershtman and Judd (1987) we use a linear demand function and we focus on the symmetric costs case. Therefore, our model is like the one they consider except for the objective function of the public firm.

The second stage equilibrium outcome is the one given by the expressions (2.7)-(2.10) once we replace c_1 by $\alpha_1 c_1$.

The reaction function in incentives of the owner of the private firm is:

$$R_1(\alpha_0) = \frac{\frac{A}{c}(1+h)(2-h)h^2 + (2-h^2)(4-h^2)}{4(2-h^2)} - \frac{h^3}{4(2-h^2)}\alpha_0. \tag{3.1}$$

Government chooses the contract that maximizes the value of net social surplus. For any value of α_1, the optimal incentive α_0 is given by:

$$R_0(\alpha_1) = \frac{(1+h)(2-h)\left((2+h) - (1+h)\frac{A}{c}\right)}{4-3h^2} - \frac{h^3}{4-3h^2}\alpha_1. \tag{3.2}$$

The slope of the reaction function is positive. In other words, incentives are

strategic complements. As in the case where both firms are private, the strategic complementarity of the market variables is transmitted to the incentive variables.

The equilibrium value for α_1 is

$$\alpha_1^N = 1 + \frac{A-c}{c} \cdot \frac{h^2\left(2-h^2\right)(1+h)}{8-8h^2+h^4}, \tag{3.3}$$

the equilibrium value for α_0 is

$$\alpha_0^N = 1 - \frac{A-c}{c}(1+h)\left(1+\frac{2h^3}{8-8h^2+h^4}\right) \tag{3.4}$$

In the Proposition below we compare the equilibrium incentives of firm 0 with the incentives the government would have set if the public firm were a monopolist. It is obtained by setting h equal to zero in (3.4). This comparison allows us to study how the incentives in public firms are adjusted after liberalization has taken place. We also compare the incentive of the public firm with the one set by the private firm to check the influence of ownership on incentives.

Proposition 4. *a) The manager of the public firm becomes more profit-oriented after liberalization. b) The private firm gives a greater weight to profits than the public firm. Nevertheless, this difference is increasing with the degree of differentiation.*

If a private firm enters the market in which the public firm is a monopoly, the incentives given to the manager of the public firm will switch from the value $\alpha_0^N(0)$ to $\alpha_0^N(h)$, where h reflects the actual degree of differentiation between the good produced by the public firm and the one produced by the private firm. After liberalization public managers become more profit-oriented because $\alpha_0^N(0) < \alpha_0^N(h)$.

As far as the comparison between the incentives of public and private managers is concerned, we have that the public firm is more aggressive than the private firm. The public firm fights the distortions due to imperfect competition by producing more than the private firm. As a consequence, we have that the price margin of the private firm is greater. We have that:

$$\frac{p_1-c}{p_2-c} = \frac{2}{-h} + h > 1$$

The difference between the incentives of public and private firms increases with the degree of differentiation, vanishing when goods become homogenous.

Therefore, as far as incentives are concerned ownership matters less the greater the competition.

4. The Temporal Dimension of Incentives

Barcena-Ruiz and Espinosa (1996) analyzed the temporal dimension of incentives in the case where firms were privately owned. The market stage is repeated twice. They consider two different ways of fixing the incentives of managers in each market stage. The first one consists in choosing the incentives for both market stages before the first one takes place (long-term contract). The second option consists in choosing the incentive for each period just before it begins (short-term contract). The crucial difference between both options is that long-term contract "makes a firms a leader in incentives" in the second market stage, "while a short-term contract makes it a follower".

We consider a reduced form of their game where owners of firms are assumed to be able directly to decide on whether to become a leader or a follower in incentives. If an owner chooses to be leader and the other to be a follower, the former chooses the incentive scheme before the latter. When both owners choose the same role, incentives are chosen simultaneously as in the previous Section. Dowrick (1986) also analyzed the outcome if firms could commit to be a leader or a follower in the market variable (either price or quantity).

The results both in Dowrick (1986) and Barcena-Ruiz and Espinosa (1996) depend on the slope of the reaction function of firms. When reaction functions are downward sloping firms prefer to be leaders, while if reaction functions are upward sloping firms prefer an asymmetric outcome. We will see that in our case we obtain a complete reverse result.

Before stating the matrix of payoffs of the game where owners of firms choose to be either leaders or followers, we carefully explain the similarities between the owner of firm 0 and the owner of firm 1. In the first place, we know for the previous Section that the reaction function in incentives of both players is upward sloping. This comes from the fact that $\frac{\partial \Pi_1}{\partial p_1 \partial p_2} > 0$ and $\frac{\partial W}{\partial p_1 \partial p_2} > 0$ (see footnote 8 in Barcena-Ruiz and Espinosa (1996)) . Both players prefer high prices when the competitor sets high prices. For the private firm this comes from the usual argument, while for the public firm comes from the fact that she likes "symmetric" outcomes due to the fact that the representative consumer cares for variety.

In spite of this similarity, one should highlight that while the private firm likes high prices, the public firm likes low prices. This will explain why we obtain

different results than Barcena-Ruiz and Espinosa (1996).

Below we write the matrix of the game played by the owners before they set the incentives. Each of them has two possible strategies: either to be a leader or be a follower.

		Public Firm	
		F	L
Pr. Firm	F	$\Pi \quad W$	$\Pi_l \quad W_l$
	L	$\Pi_f \quad W_f$	$\Pi \quad W$

where $\Pi_l < \Pi < \Pi_f$ and $W_f < W < W_l$. (see Appendix C)

We have that for both owners choosing to be a leader is a dominant strategy. In the asymmetric outcomes, we have that if the public firm is a leader she will choose an aggressive incentive scheme to trigger an aggressive reaction from the private firm. Therefore, low prices will be obtained. If the private firm is a leader she will choose a soft incentive scheme to trigger a soft reaction from the public firm, because she likes "symmetric" outcomes. Therefore, high prices will be obtained. Choosing to be a leader is a dominant strategy because it allows to be a leader if the competitor chooses to be a follower and avoids being a follower if the competitor chooses to be a leader.

>From Barcena-Ruiz and Espinosa (1996) we know that if both firms were private, the equilibrium of the game would be asymmetric: one firm would choose to be leader and the other would act as the follower. Firms obtain greater profits and set higher prices than in the case in which firms choose incentives simultaneously. Thus, the possibility of choosing roles works as a collusive device. This does not occur when one of the firms is public. Therefore, public ownership can be used to prevent collusion.

5. Final Comments

We have seen that the need for intervention is decreasing with the level of competition in the market. This result has been obtained assuming that the subsidy to the public firm is observable. This assumption implies that the subsidy not only affects the cost of the public firm but it has an strategic effect on the private firm. Observability was also assumed in Section 3 where the private firm chose managerial incentives. This assumption of observability has been objected, see Katz (1991). In our case, if we drop the assumption of observability, the equilibrium will be as if the private firm chooses prices to maximize profits and the public firm chooses prices to maximize welfare (the non delegating case). The optimal

subsidy is lower because there is no strategic effect, but the basic result of the paper holds: the optimal subsidy is decreasing in the level of competition.

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A. Appendix A

Proof of Proposition 1

$$\alpha^m - \alpha^* = \frac{-h(A-c_1)(4-2h^2+h^3\left(\frac{A-c_0}{A-c_1}\right))}{c_0(4-3h^2)} >$$

$$\frac{-h(A-c_1)(4-2h^2+2h^3)}{c_0(4-3h^2)} \geq 0$$

The first inequality comes from the upper bound imposed on c_1.

$$\frac{\partial \alpha^*}{\partial c_1} = \frac{4h-2h^3}{c_2(4-3h^2)} < 0$$

$$\left.\frac{\partial \alpha^*}{\partial h}\right|_{c_0=c_1=c} = \frac{2(A-c)(-8+6h^2-8h^3-3h^4+3h^5)}{c(4-3h^2)^2} < 0$$

$$\frac{\partial \nabla W}{\partial c_1} > 0. \text{ It follows from } \frac{\partial \alpha^*}{\partial c_1} < 0.$$

$$\left.\frac{\partial \nabla W}{\partial h}\right|_{c_0=c_1=c} = \frac{(1+h)^2(32-24h-4h^2+6h^3-13h^4+6h^5)}{(2+h)^3(4-4h-3h^2+3h^3)^2} > 0$$

B. Appendix B

$$\alpha_0^N - \alpha^m = \frac{-h(A-c)(8-6h^2+2h^3+h^4)}{c(8-8h^2+h^4)} > 0$$

$$\alpha_1^N - \alpha_0^N = \frac{2(A-c)(2+h-h^2)^2}{c(8-8h^2+h^4)} > 0$$

$$\frac{\partial(\alpha_1^N - \alpha_0^N)}{\partial h} = \frac{4(A-c)(16+8h-8h^2+8h^3+2h^4-5h^5+h^6)}{c(8-8h^2+h^4)^2} > 0$$

C. Appendix C

Definition 1. *Let us denote as* $\left(\alpha_0^N, \alpha_1^N\right)$ *the equilibrium incentives when firms choose simultaneously their managers' incentives. The pair* $\left(\alpha_0^L, \alpha_1^L\right)$ *denotes the equilibrium incentives when the public firm chooses first the incentives scheme.* $\left(\alpha_0^F, \alpha_1^F\right)$ *stands for the equilibrum incentives when the public firm behaves as the follower, and* $\left(\alpha^P, \alpha^P\right)$ *are the equilibrium incentives if both firms were private.*

Result 1 $\alpha_j^L < \alpha_j^N$, $j = 0, 1$.

α_0^L satisfies

$$\frac{\partial W\left(\alpha_0^L, R_1\left(\alpha_0^L\right)\right)}{\partial \alpha_1} \frac{dR_1\left(\alpha_0\right)}{d\alpha_0} + \frac{\partial W\left(\alpha_0^L, R_1\left(\alpha_0^L\right)\right)}{\partial \alpha_0} = 0. \tag{C.1}$$

This first derivative evaluated at $\left(\alpha_0^N, R_1\left(\alpha_0^N\right)\right)$ is negative, since its second component is zero and its first component is negative at that point (the private firm is more conservative than what the government would choose it to be). Since $W\left(\alpha_0, R_1\left(\alpha_0\right)\right)$ is a strictly concave function of α_0, it follows that $\alpha_0^L < \alpha_0^N$. Since the reaction function in incentives is upward sloping we obtain that $\alpha_1^L < \alpha_1^N$.

Result 2 $\alpha_1^N < \alpha_1^F$ and $\alpha_0^N < \alpha_0^F < \alpha_1^F$.

α_1^F satisfies

$$\frac{\partial \Pi\left(R_0\left(\alpha_1^F\right), \alpha_1^F\right)}{\partial \alpha_1} + \frac{dR_0\left(\alpha_1\right)}{d\alpha_1} \frac{\partial \Pi\left(R_0\left(\alpha_1^F\right), \alpha_1^F\right)}{\partial \alpha_0} = 0.$$

This first derivative evaluated at $\left(\alpha_0^N, R_1\left(\alpha_0^N\right)\right)$ is positive, since its first componen is zero and its second component is positive at that point (the public firm is softer than what the private firm would like it to be). Since $\Pi\left(\alpha_0, R_1\left(\alpha_0\right)\right)$ is a strictly concave function of α_1, it follows that $\alpha_1^F > \alpha_1^N$. Since the reaction function in incentives is upward sloping, we obtain that $\alpha_0^F > \alpha_0^N$.

Result 3 $W\left(\alpha_0^F, \alpha_1^F\right) < W\left(\alpha_0^N, \alpha_1^N\right) < W\left(\alpha_0^L, \alpha_1^L\right)$.

The function $W(R_0(\alpha_1), \alpha_1)$ is strictly concave function of α_1 and it is maximized at α^w. In that case, firm 1 would behave as a public firm. Thus, in equilibrum, incentives would be set so that prices equal marginal cost. Therefore, for any $\alpha_1 > \alpha^w$ welfare will be lower. Since $\alpha_1^F > \alpha_1^N > \alpha^w$, the first inequality follows. The second one follows trivially.

Result 4 $\Pi\left(\alpha_0^L, \alpha_1^L\right) < \Pi\left(\alpha_0^N, \alpha_1^N\right) < \Pi\left(\alpha_0^F, \alpha_1^F\right)$.

The function $\Pi\left(\alpha_0, R_1(\alpha_0)\right)$ is strictly increasing in α_0. The private firm would like the public firm to be as soft as possible. The first inequality follows from Result 2. The second inequality is trivial.