

## Divisionalization in Vertical Structures

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*Abstract:* We study the incentives to create divisions by a firm once it is taken into account the vertical structure of an industry. Downstream firms, that must buy an essential input to upstream firms, may create divisions. Divisionalization reduces their bargaining power against upstream firms. This effect must be weighted against the usual incentive to divisionalize, namely the increase in the share of the final market that a firm obtains through it. We show that incentives to divisionalize are severely reduced when compared with the standard results, and that even sometimes firms choose not to divisionalize at all. The paper also shows the implications of the former analysis on the internal organization of firms and on the incentives to vertically integrate.

*JEL classification:* L10, L11, L20

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## 1. Introduction

The literature on oligopoly theory has investigated the incentives of firms to create divisions that compete independently in a market. Corchón (1991), Baye, Crocker and Ju (1996) and Corchón and González-Maestre (2000) have obtained the counterintuitive result that, as divisionalization costs tend to zero, Cournot competition leads to the perfectly competitive outcome, i.e. to the full dissipation of the oligopoly rents, even when there are only two firms in the industry.

There are several ways to obtain different equilibria from the perfectly competitive outcome even with zero fixed cost per division. In González-Maestre (2000) firms are able to delegate output decisions to division managers and shape their behavior through incentives schemes. In Huck, Konrad and Müller (2001) firms compete in a contest (instead that à la Cournot). In both papers, incentives to divisionalize are reduced and output is lower than the competitive.

The present work reevaluates the incentives to divisionalize when one takes into account the vertical structure of an industry. Retail firms must buy a basic input from upstream firms. We assume unobservable contracts between upstream and downstream firms as in Rey and Tirole (1999), and two suppliers of the input, one of them more efficient than the other. Downstream firms may create divisions without any fixed set-up cost.

In the literature of divisionalization, firms have a strategic incentive to divisionalize: the creation of independent divisions commits a firm to a more aggressive behavior that increases its market share at the expense of rivals. In a vertical relationship, however, there is a countervailing effect: divisionalization reduces their bargaining power against upstream firms. We show below that in a vertical structure incentives to divisionalize are drastically reduced. In a downstream duopoly, the equilibrium number of divisions is always finite and thus the perfectly competitive outcome is never attained. Even in some circumstances firms do not divisionalize at all.

The explicit modelization of the vertical structure allows us to analyze the connection between upstream and downstream markets. When competition upstream increases, downstream firms obtain better deals from suppliers. Then the vertical relationship loses importance and downstream firms focus on the advantages of divisionalization to gain market share in the final market. Therefore, the more competitive the upstream segment of the industry, the more competitive is also the downstream segment. This result is in accordance with the evolution of different industries, for example in the US food sector, where there has been a parallel process of consolidation in manufacturing and retailing (Sexton (2000)).

Recently some other papers have recently analyzed the incentives to divisionalize with an explicit account of the vertical structure of the industry. Corts and Neher (1999) and Chemla (2000) assume, as we do, unobservable supply contracts, but they are interested in different issues. In Corts and Neher (1999), each upstream firm creates downstream divisions in order to gain a strategic advantage in the final market against rivals. They show that supply contracts, even if unobservable, may have strategic effects as long as downstream divisions are completely independent. Chemla (2000) shows that an upstream firm may encourage the entry of new downstream firms in order to reduce the bargaining power of the downstream sector of the industry. Both Corts and Neher (1999) and Chemla (2000) study, thus, the incentives of upstream firms to strengthen competition downstream. Our paper is concerned, instead, with the incentives of downstream firms to increase competition in order to gain a strategic advantage against rivals.

The rest of the paper is organized as follows. The next section briefly revisits the framework of unobservable contracts we use to model the vertical relationship between firms, and we emphasize the aspects of the model that are crucial for our analysis. Each division not only sets independently its output but also contracts the input independently from other divisions of the same firm. In this section, we obtain the main result of the paper: firms create a finite number of divisions and hence final prices are above the competitive level.

In section 3 two extensions of the basic framework are analyzed. First we allow centralized bargaining with upstream firms within a downstream firm, namely the only independent choice of a division is its level of output. The analysis shows that firms prefer a decentralized structure, where divisions both choose sales and input purchases, somewhat against current opinion that favors central purchasing (Dobson and Waterson (1999)).

In this section we also analyze the possibility that the efficient upstream firm vertically integrates with one of the downstream firms. Although vertical integration allows the upstream firm to foreclose rivals, divisionalization reduces the incentives to do so, because it results in a very competitive retail segment.

Finally, we draw some conclusions and directions for future research.

## **2. The model and its main insight**

There is an upstream firm  $U$  that produces an intermediate input at marginal cost  $\underline{c} \geq 0$ . There exists also a (less efficient) alternative source for the input, a second upstream firm that produces at marginal cost  $\bar{c} > \underline{c}$ . In the downstream sector there are two firms,  $i = A, B$ , that

transform one unit of input into one unit of final product without additional costs of production.<sup>1</sup> The final product is homogeneous and its demand is given by  $P(Q) = \alpha - Q$ .

Upstream and downstream firms set vertical contracts that establish the terms under which inputs are transferred. We model this vertical relationship following the framework in Rey and Tirole (1999), where contracts are secret (or unobservable) and firms have passive conjectures. After contracts are set, competition downstream is à la Cournot.

We want to address how the process discussed above is affected by the decision of downstream firms to act through divisions. Those divisions will be independent both (i) to bargain with suppliers and (ii) to decide the level of sales in the final market. In contrast with previous work on divisionalization that concentrate on point (ii), we want to stress the importance of the interaction of both decisions. Downstream firms may create as many independent divisions without any fixed set-up cost as they find it is in their private interest.<sup>2</sup> The full game we consider has thus the following stages:

Stage 1: Downstream firms A and B decide their firm structure, namely their number of divisions,  $n_A$  and  $n_B$ .

Stage 2: Upstream firms secretly offer each division a contract; each division chooses a supplier, orders a quantity of input and pays accordingly.

Stage 3: Divisions transform input into final product and compete in the final market à la Cournot.

Notice that we assume that the level of divisionalization is chosen before supply contracts are settled. We believe that divisionalization is a decision that affects firms and market structure so that it is a decision that cannot be easily changed. Supply contract terms instead is a conduct decision easier to modify and adapt. The order of the stages in the game pretend to reflect these considerations.

Upstream firms offer two-part supply contracts. Hart and Tirole (1990) and McAfee and Schwartz (1994) show that, when supply contracts are secret and divisions have passive conjectures, the equilibrium in the final product market is unique and characterized by the Cournot quantities with  $n = n_A + n_B$  players that produce at marginal cost  $\underline{c}$ , because in

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<sup>1</sup> We could generalize the analysis to the case of more than two downstream firms and the main results would still hold. Therefore, for the sake of simplicity, we present the results for two downstream firms.

<sup>2</sup> They sell those divisions in a competitive market for firms. For instance already existing retailers may sell their product without any additional cost. This guarantees (i) that each division is an independent firm and (ii) that firms A and B extract all the surplus to be obtained in product market competition. If contracts with divisions were secret, we would have a fixed payment equal to the expected surplus of the division.

equilibrium U serves all divisions, and sets a two-part tariff that has a marginal wholesale price  $w = \underline{c}$ . Thus, in equilibrium each division produces  $q^c(n)$  and obtains profits  $\pi^c(n)$ .

Downstream firms have always the option to use the less efficient input, and produce at marginal cost  $\bar{c}$ . Competition between upstream firms drives down payments for input until downstream firms are indifferent between producing at high and low marginal costs. More specifically, the efficient firm U supplies all divisions for a fixed fee equal to  $\pi^c(n) - \text{Max}_q \{P((n-1)q^c(n) + q) - \bar{c}\}q$ , and hence each division has net profits equal to the profits it would obtain off the equilibrium path with the second source of input,<sup>3</sup>  $\pi^D(n) \equiv \text{Max}_q \{P((n-1)q^c(n) + q) - \bar{c}\}q$ . With a linear demand  $P(Q) = \alpha - Q$ , each division has

$$\text{net profits } \pi^D(n) = \frac{1}{4} \frac{1}{(n+1)^2} (\bar{c} - \underline{c})^2 \left( 2 \left( \frac{\alpha - \bar{c}}{\bar{c} - \underline{c}} \right) - (n-1) \right)^2.$$

Therefore, in order to evaluate the profits of each division, downstream firms must take into account the effect of the total number of divisions  $n$  in  $\pi^D(n)$ .<sup>4</sup> In the earliest stage of the game, each downstream firm chooses its optimal number of divisions, that is, it solves

$$\text{Max}_{n_i} \Pi^i = n_i \pi^D(n) \text{ where } n = n_i + n_j, \forall i = A, B \text{ and } i \neq j \quad (1)$$

The optimal number of divisions satisfies the first order condition (FOC) of the former problem. The marginal revenue for downstream firm of an additional division has two terms, the increase in revenues from an additional division and the reduction in profits per division due to the increase in competition in the final market:

$$\frac{\partial \Pi^i}{\partial n_i} = \pi^D + n_i \frac{\partial \pi^D}{\partial n_i} \quad (2)$$

The standard literature of divisions shows that with a linear demand the marginal revenue of divisions is equal to zero, at  $n_i = R(n_j) = n_j + 1$ . The incentives to obtain a larger share in the market are so strong that in equilibrium firms dissipate all the rents through an excessive number of divisions. In our model, the FOC (2) becomes

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<sup>3</sup> Notice that rival divisions expect an agreement between U and each division and thus produce  $q^c(n)$ .

$$\frac{\partial \Pi^i}{\partial n_i} = \pi^D + n_i \frac{\partial \pi^i}{\partial n_i} = \left\{ \left[ P((n-1)q^c(n) + q^{off}) - \bar{c} \right] + n_i P' \frac{\partial (n-1)q^c}{\partial n_i} \right\} q^{off} = 0, \quad (3)$$

where we define  $q^{off} \equiv \arg \text{Max}_q \{ P((n-1)q^c(n) + q) - \bar{c} \} q = \frac{1}{2} \frac{1}{(n+1)} (2(\alpha - \bar{c}) - (n-1)(\bar{c} - \underline{c}))^2$ , i.e.

$q^{off}$  is the production of a division off the equilibrium path. When we compare the marginal revenue of an additional division in our case and in the literature we see:

- (i) The profits of a division is now affected by the efficiency of the second source, because the rents that the upstream firm extracts to the division is increasing in  $\bar{c}$ ; this is reflected in the first term in brackets in (3), the mark-up off the equilibrium path, which is the relevant one for downstream firms.
- (ii) On the other hand, the reduction in profits per division as the number of divisions increase comes through the change in the production of rivals. This is the second term in brackets in (3). Rivals produce at low marginal costs ( $\underline{c}$ ); this effect is basically the same in our model as in any other standard paper on divisionalization.

Thus, the vertical structure of the industry reduces the positive incentives to set divisions, but does not change the negative incentives. For linear demands, when  $\bar{c}$  increases the mark-up decreases, whereas  $n_i P' \frac{\partial (n-1)q^c}{\partial n_i}$  is kept constant. Therefore increases in  $\bar{c}$  reduce the incentives to divisionalize.

Proposition 1 states the result from this discussion.

**Proposition 1.** *The unique equilibrium is given by*

$$n_A^* = n_B^* = \text{Max} \left\{ 1, \frac{1}{2} \left[ \left( 2 \left( \frac{\alpha - \bar{c}}{\bar{c} - \underline{c}} + 1 \right) \right)^{1/2} - 1 \right] \right\}.$$

*For low levels of  $\bar{c}$  but strictly greater than  $\underline{c}$ , firms divisionalize. For high levels of  $\bar{c}$  firms do not divisionalize at all. In any case, downstream firms obtain strictly positive profits.*

**Proof.** With linear demands the first order condition (3) can be written as

$$\frac{\partial \Pi^i}{\partial n_i} = 0 \Leftrightarrow 2(\alpha - \bar{c})(n_j + 1) - n_i - (\bar{c} - \underline{c})(n_i + n_j)^2 + 4n_i - 1 = 0, \quad (4)$$

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<sup>4</sup> Notice that for a sufficiently high number of divisions, each division faces a residual demand  $P((n-1)q^c(n) + q)$  such that  $P((n-1)q^c(n)) < \bar{c}$ ; in such a case the second source is irrelevant, the efficient upstream firm may reap all the rents from the vertical relationship and  $\pi^D$  is driven down to zero.

$\forall i = A, B$  and  $i \neq j$ . If we take the number of divisions as a continuum number  $n_i, n_j \geq 1$  the unique solution is the one stated in the proposition. ■

The literature of divisionalization has shown that, in an oligopoly market, the strategic incentives to increase the market share of a firm lead to the full dissipation of oligopolistic rents through the excessive creation of divisions. To avoid perfect competition as the outcome of the divisionalization process, previous papers need to impose exogenously a restriction on the number of divisions that firms may create: an ad-hoc upper bound, fixed costs of divisionalization, etc. We obtain the same result by taking into account the vertical relationships within the industry.

Notice from (4) that the optimal number of divisions  $n_i = R(n_j)$  now satisfies  $R(n_j) < n_j + 1$ . Indeed, for a sufficiently inefficient alternative, in equilibrium *downstream firms do not divisionalize at all*, but in any case *downstream firms choose a finite number of divisions*.  $R(n_j)$  approaches  $n_j + 1$  as  $\bar{c}$  approaches  $\underline{c}$ , i.e., as the difference in costs between suppliers vanishes. Thus we obtain in the limit the standard result (see Baye, Crocker and Ju (1996) and Corchón (1991)) that, when divisionalization has no fixed costs and the vertical structure of the industry is assumed away, each firm wants to set one more division than its rival, which drives them to the competitive outcome.

Another way to see it is to look at the comparative statics of the final price with respect to the differences in costs  $\bar{c} - \underline{c}$ . The final price is  $p = \frac{1}{n_A + n_B + 1} (\alpha + (n_A + n_B) \underline{c})$ ; for the number of divisions in equilibrium, the final price becomes

$$p^* = \left( \frac{(\alpha - \underline{c})(\bar{c} - \underline{c})}{2} \right)^{1/2} + \underline{c}. \quad (5)$$

The final price decreases as  $\bar{c}$  decreases, and converges to the competitive price as the cost differential vanishes. The decrease in the cost differential may be interpreted as an increase in competition upstream. Therefore we obtain that more competition upstream is finally reflected in the final price through an increase in competition downstream. Without divisionalization, as in Rey and Tirole (1999), the final price does not depend on competition upstream, because input contracts are efficient. The only effect is that rents are redistributed between suppliers and downstream firms. In our model, input contracts are still efficient, and the result is obtained through the effect of upstream rivalry in the number of divisions.

### **3 Extensions**

In recent years there has been a trend to decentralize decision making inside organizations in order to give better incentives to agents in charge of taking decisions. However, it has been recognized that fully decentralized organizations may lose some buyer power against suppliers. Then, some reports advocate for organizational forms that combine the decentralization of decision making with some degree of centralization in purchasing tasks. For instance, hospital management and operating decisions have been decentralized in many countries. Nevertheless there are proposals to centralize purchases at the global level of the health system (NHS Procurement Review, 1998). Closer to our framework, in the European retail sector some of the large buyer groups are purchasing groups that buy goods from manufacturers for independent retailers (Dobson and Waterson, 1998).

In the previous section, downstream firms are assumed to choose very decentralized structures because divisions take all decisions, including input purchases. Given the discussion above, one may believe that downstream firms could improve their performance by retaining the purchasing decisions at the firm level. However, as it is shown below, downstream firms do prefer to maintain the decentralized structure assumed in section 2.

Another issue that we do not take into account in the former section is the possibility of vertical integration between U and one of the downstream firms. Vertical integration may be valuable for U in order to foreclose the downstream segment (see Rey and Tirole (1999)). We show that vertical integration becomes much less valuable when downstream firms may divisionalize and that actually in equilibrium vertical integration never occurs.

Therefore the two extensions considered in this section support the analysis in section 2 in the sense that an enlarged model that considers the possibility of centralized structures or of vertical integration do not alter the predictions of the original model.

#### **3.1 Central purchasing**

Assume now that firms A and B may create divisions that independently set their level of production, and that they may also create a Central Office that would be in charge of bargaining supply contracts with upstream firms. This means that contractual conditions for divisions of a firm that creates a Central Office are the same for all of them. If U and the Central Office do not reach an agreement, all the divisions of the downstream firm must use the second source of input.

We consider in this section the following game.

Stage 1. Firms choose at the same time their internal structure (they create a Central Office in charge of input purchases or they decentralize completely) and the number of divisions.

Stage 2. Upstream firms offer secret contracts to downstream firms (to the central purchasing if a firm is centralized; to divisions in a decentralized structure). Downstream firms choose a supplier and order input.

Stage 3. Divisions transform the input into final product and compete in the final market.

At first sight, centralized bargaining would reduce the problem with divisionalization observed in the former section (namely that each division has a poor position when setting supply contracts). However we will see that centralized bargaining benefits more the upstream firm U than the downstream firm that creates a Central Office.

One must take into account that the efficient upstream firm recovers its commitment power against a downstream firm with a centralized structure. When U offers a contract to a Central Office, this contract is binding for all the divisions belonging to this firm, and hence the contract may be used to restrict total production. More formally, when U offers a supply contract to a downstream firm with a Central Office, it has incentives to offer a two-part tariff  $T(q) = T + wq$  with  $w > \underline{c}$ , i.e., a marginal wholesale price above the marginal cost of production, in order to induce a more collusive outcome in the final market (which allows U to reap more rents from this downstream firm).

Consider that firm A has created a Central Office and it is expected a total production  $Q_B$  from divisions of firm B. The next lemma characterizes the contract that U offers to A.

**Lemma 1.** *The upstream firm U offers a supply contract to firm A with a wholesale marginal*

*price*  $w_A = \text{Min} \left\{ \frac{1}{2n_A} ((n_A - 1)(\alpha - Q_B) + (n_A + 1)\underline{c}), \bar{c} \right\}$  *and a fixed fee per division*

$T_A = \text{Max} \{ \pi(w_A) - \pi(\bar{c}), 0 \}$ .

**Proof.** See the appendix. ■

Note that  $w_A$  is equal to  $\underline{c}$  only if  $n_A = 1$ . When  $w_A < \bar{c}$ , firm A produces just the same as if it had set just one division. In other words, the upstream firm may countervail the divisionalization effect on the final market through the marginal wholesale price, which allows U to increase the rents obtained from A through the fixed fee.

Lemma 2 then comes easily.

**Lemma 2.** *For levels of divisionalization such that  $w_A < \bar{c}$  the optimal number of divisions for firm A is  $n_A = 1$ , i.e., not to divisionalize at all.*

**Proof.** Given that divisions of firm B produce a total amount  $Q_B$  (they react as if firm A had just one division), total net profits of firm A are<sup>5</sup>

$$\Pi^A = n_A \pi(\bar{c}) = \frac{n_A}{(n_A + 1)^2} (\alpha - Q_B - \bar{c})^2, \quad (6)$$

which is a decreasing function of  $n_A$ . ■

For levels of divisionalization that leads to  $w_A < \bar{c}$ , firm A is unable to increase final production in equilibrium. Net profits of firm A are the profits obtained off the equilibrium path, when all its divisions must use the second source of the input. But these profits decrease in the number of division (as more divisions would imply a more competitive final market off the equilibrium path), and hence the bargaining power of firm A is decreasing in the number of divisions.

Only when  $w_A = \bar{c}$ , firm A produces more than if he had just one division, and thus divisionalization has the usual strategic effect on the final market. The next lemma states the implications of a central purchasing office on the number of divisions that firm A sets.

**Lemma 3.** *If firm A chooses a centralized structure, then its optimal number of divisions is either  $n_A = n_B + 1$  or  $n_A = 1$ . For  $n_A = n_B + 1$  to be optimal it is necessary that  $w_A = \bar{c}$  for such a level of divisionalization.*

**Proof.** See the appendix. ■

Notice that with linear demands a firm with centralized purchasing and  $w = \bar{c}$  has just the same incentives to divisionalize than in the standard literature of divisionalization. This leads to the following lemma.

**Lemma 4.** *For downstream firms, it is always better to decentralize.*

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<sup>5</sup> Recall that these are the profits A obtains if all its divisions produce with the alternative input at marginal cost  $\bar{c}$ .

**Proof.** We analyze without loss of generality the behavior of firm A.

Assume firm B chooses to decentralize and a given number of divisions  $n_B$ . Note that, if A chooses to centralize and to set a number of divisions  $n_A = 1$ , then it would obtain at least the same profits with a decentralized structure. On the other hand, profits under a centralized structure for firm A reaches at  $n_A = n_B + 1$  at most a maximum of

$$\Pi^A = \frac{1}{4} \frac{1}{n_B + 1} \left[ (\alpha - \bar{c}) - n_B (\bar{c} - \underline{c}) \right]^2 \quad (\text{and profits are lower if } w_A < \bar{c} \text{ at } n_A = n_B + 1 \text{ and a}$$

higher number of divisions are required for  $w_A = \bar{c}$ ). But these profits may be attained also under a decentralized structure with the same number of divisions, and they may be even higher with the optimal number of divisions under a decentralized structure  $n_A = R(n_B) < n_B + 1$ .

Assume now that firm B chooses to centralize and a given number of divisions  $n_B$ . Note again that, if firm A chooses to centralize and to set a number of divisions  $n_A = 1$ , then it would obtain at least the same profits with a decentralized structure. On the other hand, profits under a centralized structure for firm A reach at most a maximum at  $n_A = n_B + 1$  of

$$\Pi^A = \frac{1}{4} \frac{1}{n_B + 1} (\alpha - \bar{c})^2 \quad (\text{and are lower if } w_A < \bar{c} \text{ at } n_A = n_B + 1 \text{ and a higher number of}$$

divisions are required for  $w_A = \bar{c}$ ). But firm A may attain also these profits under a decentralized structure with the same number of divisions, and they may be even higher with the optimal number of divisions under a decentralized structure  $n_A = R(n_B) < n_B + 1$ . ■

From lemma 4 it is immediate the following proposition.

**Proposition 2.** *Both firms choose a decentralized structure and the same number of divisions*

$$n_A^* = n_B^* = \text{Max} \left\{ 1, \frac{1}{2} \left[ \left( 2 \left( \frac{\alpha - \bar{c}}{\bar{c} - \underline{c}} + 1 \right) \right)^{\frac{1}{2}} - 1 \right] \right\}.$$

In section 2, we assume that the creation of divisions implies to decentralize all the relevant decisions. In our framework, where we consider a vertical structure, this means that divisions decide on sales and supplies. Here, downstream firms have more flexibility in the assignment of tasks to divisions. In particular, they may create a Central Office in charge of input purchases. Proposition 2 shows that anyway downstream firms prefer a completely decentralized structure. Hence divisionalization, as usually interpreted in the literature, arises as an equilibrium outcome.

### 3.2 The incentives to vertically integrate.

We next study the incentives of the efficient upstream firm to vertically integrate with one of the downstream firms. If firms U and A vertically integrate, the resulting firm U/A decides not only the number of divisions it creates but also the number of divisions that are centrally controlled.<sup>6</sup> As in section 2, the incentive to create independent divisions is to gain market share at the expense of competitors. To maintain some divisions under control (i.e. the possibility to serve the final good directly to consumers) allows U/A to foreclose rivals in the input market. We show below that both decisions are intertwined.

More precisely, the timing of decisions that we consider is the following:

Stage 1: The efficient upstream firm and retail firm A decide whether to integrate or not.

Stage 2: Firms decide their number of independent divisions.

Stage 3: The vertically integrated firm decides the number of centrally controlled divisions.

Stage 4: Upstream firms secretly offer each independent division a tariff; each division then orders a quantity of input to each supplier and pays accordingly.

Stage 5: Divisions transform input into final product and compete in the final market à la Cournot.

If at stage 1 there is no vertical integration, the analysis is the same that in section 2. So now we focus on the case in which U and A vertically integrate.

In the last stage, divisions compete in quantities. Divisions under control of firm U/A will maximize joint profits and thus will behave just as one division. Thus we may assume without loss of generality that U/A sets only one division under its control, if any.

In stage 4, U/A provides internally the input to the controlled division. Further it must decide whether to serve the input to the remaining divisions. Firm U/A does not distinguish the independent divisions it has created and divisions created by firm B. Firm U/A would like to restrict the production of the remaining divisions as much as possible in order to reduce competition in the final market. Since they can obtain the input from the inefficient upstream firm at wholesale price  $w = \bar{c}$ , firm U/A finally serves the input to them at such a price.<sup>7</sup> Therefore there is Cournot competition in the final product market with the division under control of firm U/A producing at marginal cost  $\underline{c}$  and the remaining divisions producing at

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<sup>6</sup> This kind of decisions are relevant empirically. Scott (1995) shows that is it usual that firms have both centrally controlled and independent divisions.

<sup>7</sup> This is shown in Hart and Tirole (1990) for the duopoly case.

marginal cost  $\bar{c}$ . If U/A does not control one division, supply contracts between U/A and independent divisions are as in section 2.

In the third stage, U/A must decide whether to create a centrally controlled division. Denote as  $n_A$  and  $n_B$  the number of independent divisions created by firm U/A and firm B, respectively, in stage 2. When U/A controls one division, its profits are

$$\pi^{U/A} + (n_A + n_B)(\bar{c} - \underline{c})q(\bar{c}), \quad (7)$$

where  $\pi^{U/A}$  are the profits of the controlled division and  $q(\bar{c})$  is the input that each independent division buys. On the other hand, when U/A does not control a division, its profits are as in section two, i.e.

$$(n_A + n_B)\left\{\pi^c(n_A + n_B) - \pi^D(n_A + n_B)\right\}, \quad (8)$$

where as in section 2  $\pi^c(n_A + n_B)$  are the Cournot profits when all divisions have marginal costs  $\underline{c}$ , and  $\pi^D(n_A + n_B)$  are the profits divisions would obtain off the equilibrium path with the second source.

**Lemma 5.** *For any level of divisionalization  $(n_A, n_B)$ , the vertically integrated firm prefers to create a centrally controlled division.*

**Proof.** See the appendix. ■

When firms decide their number of independent divisions in stage 2, according to lemma 5, they expect partial vertical foreclosure from U/A, through the creation of a centrally controlled firm. Therefore U/A sets  $n_A$  to maximize

$$\Pi^{U/A} = \pi^{U/A} + (n_A + n_B)(\bar{c} - \underline{c})q(\bar{c}) + n_A\pi(\bar{c}), \quad (9)$$

and B sets  $n_B$  in order to maximize

$$\Pi^B = n_B\pi(\bar{c}), \quad (10)$$

where  $\pi(\bar{c})$  are the profits that independent divisions expect to obtain in the last stage, and are equal to the Cournot profits when  $n_A + n_B$  firms produce at marginal costs  $\bar{c}$  and one firm (the division under U/A's direct control) produces at cost  $\underline{c}$ .

Vertical foreclosure by firm U/A implies that firm B has the standard strategic incentives to divisionalize, because independent divisions are served at a constant wholesale price  $\bar{c}$ . In opposition to what happened in section 2, the number of divisions does not affect the supply contracts that are going to be signed and therefore the countervailing force to limit the number of divisions disappears.

For firm U/A, matters are more complicated. Firm U/A obtains market revenues from the centrally controlled division and from selling the input to independent divisions,  $\pi^{U/A} + (n_A + n_B)(\bar{c} - \underline{c})q(\bar{c})$ . Moreover it obtains profits from selling independent divisions,  $n_A\pi(\bar{c})$ . The former profits decrease in the number of divisions because it induces more competition in the final market. But U/A erodes market share to B through the creation of divisions, and hence has incentives to create them. The following lemma shows which is the final outcome of this trade-off.

**Lemma 6.** *The optimal number of independent divisions for the vertically integrated firm is  $n_A = R(n_B) = n_B$ , and the optimal number of divisions for firm B is  $n_B = R(n_A) = n_A + 2$ . Then in equilibrium the final price is  $\bar{c}$ .*

**Proof.** See the appendix. ■

The efficient upstream firm would vertically integrate with firm A, as Hart and Tirole (1990) already noted and lemma 5 establishes in our model, to foreclosure firm B and reduce in this way competition downstream. But this tactic backfires because it has the side effect to promote the creation of divisions, to the point that vertical integration and divisionalization lead to perfect competition in equilibrium. In the final market there is a firm (the division that is still under U/A's control) that produces at marginal cost  $\underline{c}$  the quantity  $q = \bar{c} - \underline{c}$ , whereas the remaining firms produce an aggregate level of production  $(\alpha - \bar{c}) - (\bar{c} - \underline{c})$ . The integrated firm obtains total profits (profits of its divisions plus profits from selling the input to the industry)  $\Pi^{U/A} = (\bar{c} - \underline{c})(\alpha - \bar{c})$ .

Now, it is worth to vertically integrate? Without vertical integration, U is unable to commit to high input prices. The vertically integrated firm U/A, instead, may commit to foreclosure

rivals. But divisionalization has the side effect to create a very competitive market through the creation of independent divisions.

The following proposition compares joint profits when U and firm A do not vertically integrate (i.e. the profits they obtain in section 2) with profits when they do integrate.

**Proposition 3.** *Firms U and A obtain more profits when they do not integrate.*

**Proof.** When U and A do not integrate, the number of divisions, from proposition 1, are

$$n_A^* = n_B^* = \text{Max} \left\{ 1, \frac{1}{2} \left[ \left( 2 \left( \frac{\alpha - \bar{c}}{\bar{c} - \underline{c}} + 1 \right) \right)^{\frac{1}{2}} - 1 \right] \right\}. \text{ Their profits are } \Pi^U = n(\pi^c(n) - \pi^D(n)) \text{ and}$$

$\Pi^A = n_A^* \pi^D(n)$  respectively, where  $n = n_A^* + n_B^*$ . If they do integrate, their joint profits are

$\Pi^{U/A} = (\bar{c} - \underline{c})(\alpha - \bar{c})$ . Simple computation shows that joint profits are higher without

$$\text{integration, } \Pi^U + \Pi^A - \Pi^{U/A} = \frac{1}{8} \left[ \left( 2 \left( \frac{\alpha - \bar{c}}{\bar{c} - \underline{c}} + 1 \right) \right)^{\frac{1}{2}} - 3 \right] (\bar{c} - \underline{c}) (2(\alpha - \bar{c}) - (\bar{c} - \underline{c})) > 0, \text{ as}$$

long as  $n_A^* = n_B^* > 1$ . ■

From proposition 3 one may conclude that vertical integration will not be observed in our setup. Divisionalization changes the usual wisdom that upstream firms have incentives to vertically integrate to foreclosure rivals. Notice that consumers would clearly benefit from vertical integration, as firms would have strong incentives to divisionalize and this would lead to a very competitive downstream industry. But this is just why vertical integration does not happen: only the division under U/A's control would obtain strictly positive profits. The profits of the remaining downstream divisions would be completely dissipated. A disintegrated industry, instead, and as we have shown in section 2, implies lower incentives to divisionalize, and hence higher profits in the industry as a whole.

#### 4. Concluding remarks

This paper has analyzed the incentives of downstream firms to create divisions when we take into account the vertical structure of an industry. We show that firms divisionalize less than what was suggested in previous related work. Excessive divisionalization reduces the bargaining power against upstream firms, and this effect countervails the usual strategic incentive to divisionalize in order to gain market share in the final market.

In a first analysis, we have assumed that downstream firms were forced to decentralize all decisions to divisions. In our framework, where we consider a vertical structure, this means that divisions decide on output and procurement. We have analyzed then the possibility that downstream firms may create less decentralized internal structures, where a central office is in charge of input purchases. We have shown that downstream firms prefer the decentralized structure. Hence divisionalization, interpreted as a decentralized structure, emerges as an endogenous decision. We have also considered vertical integration as a mechanism to foreclose the downstream segment. We show that vertical integration becomes much less valuable when downstream firms may divisionalize.

We have shown in this paper that the level of competition upstream is reflected in the level of competition downstream. For products for which upstream firms were able to extract more rents from the downstream segment, it should be observed fewer retailers selling the good. Thus, our results suggest a stylized fact that could be analyzed empirically in future research.

## Appendix

**Proof of lemma 1.** The upstream firm chooses the contract (per division)  $T(q) = T_A + w_A q$ , given an expectation on the level of production  $Q_B$ , that solves the following problem:

$$\underset{\{T_A, w_A\}}{Max} n_A \{ \pi(w_A) - T_A + (w_A - \underline{c})q(w_A) \} \text{ subject to } w_A \leq \bar{c} \text{ and } \pi(w_A) - T_A \geq \pi(\bar{c}),$$

that amounts to solve the problem

$$\underset{w_A}{Max} n_A \left\{ \frac{1}{(n_A + 1)^2} (\alpha - Q_B - w_A)^2 + (w_A - \underline{c}) \frac{1}{n_A + 1} (\alpha - Q_B - w_A) \right\} \text{ subject to } w_A \leq \bar{c}.$$

Then we obtain the values for  $w_A$  stated in the lemma. ■

**Proof of lemma 3.** Assume that the level of divisions of firm A,  $n_A$ , is sufficiently high for  $w_A = \bar{c}$ . If firm B has settled a decentralized structure, in equilibrium, we have  $n_A$  firms that produce at cost  $\bar{c}$  and  $n_B$  firms that produce at cost  $\underline{c}$ . Each division of firm A obtains a profit

$$\pi_A = \frac{1}{(n_A + n_B + 1)^2} \left( (\alpha - \bar{c}) - n_B (\bar{c} - \underline{c}) \right)^2, \text{ and total profits } \Pi^A = n_A \pi_A \text{ reach a maximum}$$

at  $n_A = n_B + 1$ . If firm B sets also has settled a centralized structure, then in equilibrium all firms

$$\text{work at cost } \bar{c}, \text{ each division of firm A obtains now a profit } \pi_A = \frac{1}{(n_A + n_B + 1)^2} (\alpha - \bar{c})^2,$$

and again total profits  $\Pi^A = n_A \pi_A$  reach a maximum at  $n_A = n_B + 1$ .

For a given structure of firm B, if at  $n_A = n_B + 1$  the upstream firm sets  $w_A < \bar{c}$ , then profits for firm B are strictly higher at  $n_A = 1$  than for any  $n_A > n_B + 1$  for which  $w_A = \bar{c}$ : For the minimum  $n_A$  such that  $w_A = \bar{c}$ , divisions of firm B would set the same level of production  $Q_B$ , and from lemma 2 firm A would obtain higher profits with just one division. And furthermore profits are decreasing in the number of divisions  $n_A$  as long as  $n_A = n_B + 1$ . ■

**Proof of lemma 5.** The vertically integrated firm must compare profits when it controls a division,

$$\frac{1}{(n_A + n_B + 2)^2} \left( (\alpha - \bar{c}) + (n_A + n_B + 1)(\bar{c} - \underline{c}) \right)^2 + (n_A + n_B)(\bar{c} - \underline{c}) \frac{1}{n_A + n_B + 2} \left( (\alpha - \bar{c}) - (\bar{c} - \underline{c}) \right)$$

with profits if it leaves out the retail stage (all its divisions are independent),

$$(n_A + n_B) \left\{ \frac{1}{(n_A + n_B + 1)^2} (\alpha - \underline{c})^2 - \frac{1}{4(n_A + n_B + 1)^2} \left( 2(\alpha - \bar{c}) - (n_A + n_B - 1)(\bar{c} - \underline{c}) \right)^2 \right\}.$$

Simple computation shows that profits are higher in the first case. ■

**Proof of lemma 6.** The integrated firm chooses the number of divisions  $n_A$  that maximize

$$\begin{aligned} & \frac{1}{(n_A + n_B + 2)^2} \left( (\alpha - \bar{c}) + (n_A + n_B + 1)(\bar{c} - \underline{c}) \right)^2 + (n_A + n_B)(\bar{c} - \underline{c}) \frac{1}{n_A + n_B + 2} \left( (\alpha - \bar{c}) - (\bar{c} - \underline{c}) \right) + \\ & + n_A \frac{1}{(n_A + n_B + 2)^2} \left( (\alpha - \bar{c}) - (\bar{c} - \underline{c}) \right)^2 \end{aligned}$$

where the first term are the profits of its retail subsidiary (the division still owned by the vertically integrated firm), the second term are the profits obtained selling the input, and the third term are the profits obtained from creating  $n_A$  independent divisions. The optimal number of divisions for the vertically integrated firm is  $n_A = R(n_B) = n_B$ . For firm B, the optimal number of divisions maximizes

$$n_B \frac{1}{(n_A + n_B + 2)^2} \left( (\alpha - \bar{c}) - (\bar{c} - \underline{c}) \right)^2$$

and it is  $n_B = R(n_A) = n_A + 2$ . ■

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