Partial price discrimination by an upstream monopolist*

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November 15, 2006

Abstract

We analyze third degree price discrimination by an upstream monopolist to a continuum of heterogeneous downstream firms. The novelty of our approach is to recognize that customizing prices may be costly, which introduces an interesting trade-off. As a consequence, partial price discrimination arises in equilibrium. In particular, we show that inefficient downstream firms receive personalized prices whereas efficient firms are charged a uniform price. The extreme cases of complete price discrimination and uniform price arise in our setting as particular cases, depending on the cost of customizing prices.

J.E.L codes: D4, L11, L12

Keywords: Price discrimination, input markets

* We thank Aleix Calveras, Daniel Cardona-Coll, María Paz Espinosa, Angel Hernando, Inés Macho-Stadler, David Pérez-Castrillo and Juana Santamaría. Financial support from 9/UPV 00035.321-13560/2001, SEJ 2004-02172, BEC2001-0535, GV01-391, SEJ2004-07530-C04-04 and the IVIE is gratefully acknowledged.

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1. Introduction

In this paper, we analyze third degree price discrimination by an upstream monopolist. Third degree price discrimination can be defined as the possibility to charge different linear prices to different (groups of) customers. In order for price discrimination to be feasible, it must be possible to separate different (groups of) customers, which is called market segmentation. The seller must also be able to keep resales from occurring. There has been a long debate on the competitive effects of price discrimination. The Robinson-Patman Act, for example, makes it unlawful to discriminate in price between different purchasers if the effect of the discrimination may substantially lessen competition or create a monopoly.

Many papers, have studied the welfare effects of price discrimination. For the case of final good markets we can mention, among others, Robinson (1933), Schmalensee (1981), Varian (1985) and Schwartz (1990). However, as Yoshida (2000) recognizes "the vast majority of legal and other policy disputes over price discrimination concern input markets, not final good markets". Then, it seems important the analysis of cases where the discriminating monopolist is an input supplier and the buyers are downstream firms producing a final good. In this setting, Katz (1987) and DeGraba (1990) show that price discrimination lowers welfare, because low cost downstream firms are charged higher prices. Total output, however, does not change as a consequence of price discrimination. Yoshida (2000) constructs a model where total output does change and obtains the counterintuitive result that an increase in total output is a sufficient condition for a welfare decrease.

In a recent paper, Liu and Serfes (2004) introduce a possible limitation to the extent of price discrimination. They study a setting where firms can acquire costly consumer information that allows them to refine market segmentation. It is shown that firms only invest when the quality of information is good enough.

In the present paper, we study another possible limitation to price discrimination, namely, the fact that customizing prices is costly. This introduces an interesting trade-off in the analysis: the gains of price discrimination have to be compared with its costs, which allows us to endogenously determine the extent of price discrimination. Notice that our paper is close in spirit to Liu and Serfes (2004) in the sense that, in both papers, price discrimination involves a costly investment that imposes a trade-off on the decision to price discriminate. However, whereas Liu and Serfes (2004) focus on final good markets we investigate intermediate markets. Moreover, the nature of the investment is very different in the two papers. Whereas in Liu and Serfes (2004) it allows to improve market segmentation, in our paper it allows to customize prices. To the best of our knowledge, this possibility has been neglected so far by the literature.

We consider an upstream monopolist selling an input to a continuum of downstream firms producing an homogeneous good. We assume that downstream firms are heterogeneous in their production cost. Transactions between the upstream supplier and downstream firms can occur either at a common posted price or at a personalized price. The latter option requires a costly specific investment in the form of a link that allows the upstream firm to adjust the supply contract to the individual characteristics of the firms[1]. In other words, the creation of links allows the upstream firm to price discriminate among its linked customers. Price discrimination is profitable because firms' differences in costs translate into different elasticity of input demands. In particular, it is the case that the higher the cost of a downstream firm the higher its input demand elasticity. Therefore, the upstream firm would like to adjust upwards the wholesale price for low cost firms and downwards for high cost firms, knowing that the personalized contract will only be accepted if it offers a discount with respect to the posted price.

Regarding the creation of links, we will analyze two possible cases: on the one hand, the links are created by the upstream firm in a centralized way; on the other hand, each downstream firm decides whether to establish a link with the upstream firm.

In the first case, the upstream firm prefers to connect high cost firms, because low cost firms would reject the personalized contract whenever the posted price market exists. In the second case, we have that the gains of creating a link for downstream firms are increasing

[1] This cost can represent an investment in a technology that allows for personal communication. It can also include the direct costs associated with tailoring and enforcing a large number of contracts (Lafontaine and Oxley (2004)). Or it could also arise as a fee that an intermediary firm charges to connect buyers and sellers.

in their costs, because the higher their costs the higher the discount they will receive in the personalized contract. This explains that, again, market is segmented such that high cost firms create links and are treated personally and low cost firms attend the posted price market.

In both cases, in equilibrium, some firms receive a personalized price while others are supplied at a common price. This is what we call partial price discrimination. The extreme cases of complete price discrimination and uniform pricing, studied in the earlier literature, arise in our setting as particular cases when the cost of the link vanishes and when it is large enough respectively.

Regarding the effect of (partial) price discrimination on social welfare, things are simplified because we get the result that total output does not depend on the distribution of links. Then, (partial) price discrimination only affects total production costs. Given that the upstream firm subsidizes inefficient firms through price discrimination, total cost increases, which reduces social welfare. Therefore, in our context, we could prescribe not to allow for price discrimination.

In the last section of the paper, we apply the model to a case where the links are provided by an intermediary firm. We can interpret the intermediary as a Business-to-Business (B2B) firm that allows for online communications and transactions between buyers and sellers in exchange for a per-transaction fee. We consider the case of a non-industry participant.[2] Observe that e-commerce is a good illustration of our model, given that price discrimination is a common practice in the Internet. For example, one implication that has already been noticed in the business press is that the extent of the information obtained in Internet opens new possibilities for firms to price discriminate. One very important information that sellers can obtain come from the past purchase record of their customers ("it [Safeway] uses its website for (...) collecting and mining data on consumer's preferences both from the site and from loyalty cards, so it can personalize promotions" (The Economist, June 24th 1999).

[2]In practice, the fast growth of the e-commerce has induced also large firms to organize their own B2B to manage their relationships with customers and suppliers (Milliou and Petrakis, 2004). For example, in 1999, Ford and General Motors announced that their huge purchasing operations would be transferred to the web.

Another advantage of Internet is that as communication is personal, price cuts are only observed by targeted sellers. The following quotation of the FTC Report on "Competition Policy in the world of B2B Electronic Marketplaces" clarifies the situation "(...) sellers can customize price lists to reflect agreements reached with specific buyers but ensure that those prices can be viewed only by the intended buyers".

In the paper, we consider that a link between the upstream firm and a particular downstream firm is created only when both of them pay a subscription fee charged by the intermediary firm. We obtain partial price discrimination as the equilibrium outcome of an extended game where the cost of creating a link is endogenously chosen by an intermediary firm. We get, as before, that high cost firms are the ones treated personally. Moreover, the burden of the (endogenous) cost of price discrimination mainly falls over the downstream side of the market.

The rest of the paper is organized as follows. In the following section, we present the general model and solve it for the cases where the upstream firm and downstream firms choose the links respectively. In Section 3, we apply the model to the case where the links are provided by an intermediary firm. Finally, the last section discusses the results and opens new avenues for future research.

2. The Model

We assume that there is an upstream monopolist producing an intermediate good at no cost. There also exists a continuum of downstream firms that transform this input on a one-for-one basis into a final homogeneous good. The cost of this transformation for downstream firm i is given by:

$$C_i(q_i) = c_i q_i + q_i^2.$$

Downstream firms are heterogeneous in parameter c_i, which is assumed to be uniformly distributed among them in the interval [0,1]. Market demand is given by $P(Q) = A - Q$.

The timing of the game is as follows:

In the first stage, the links are created. We will consider two different possibilities, namely, either they are chosen in a centralized way by the upstream firm or they are decided individually by downstream firms. In any case, the cost of creating a link is f.

In the second stage, the upstream firm decides a uniform wholesale price w to supply the input to the firms attending the posted price market and an individual wholesale price of the form $w_i = b - ac_i$ to be offered to each linked downstream firm, where a and b are parameters to be chosen by the upstream firm.

In the third stage, downstream firms decide how many units to buy from the upstream firm and how many units to sell to final consumers. We allow linked firms to attend the posted price market and, therefore, they will make use of the personalized contracts only when $w \geq b - ac_i$.

We look for the Subgame Perfect Nash Equilibrium of the game solving it by backward induction.

Solving explicitly this game is complex. Moreover, as we show below, in order to obtain the equilibrium distribution of links, which is our main interest, it is enough to solve a simplified version of the game (called Game I) where linked firms do not have the option to be supplied in the posted price market. Very nicely, we will show that there is a strong relationship between the equilibria of Game I and the equilibria of our original game.

In the third stage, as we have a continuum of firms, they behave as price taking firms. On the one hand, linked firm i chooses output q_i to maximize its profits:

$$\pi_i = Pq_i - c_i q_i - q_i^2 - w_i q_i.$$

This leads to the following individual supply function for linked firm i:

$$S_i(P) = \frac{P - c_i - w_i}{2}.$$

On the other hand, the individual supply of a non-linked firm j is similarly obtained and amounts to:

$$S_j(P) = \frac{P - c_j - w}{2}.$$

The market clearing condition is given by:

$$\int_N \left(\frac{P - c_i - w}{2}\right) dc_i + \int_L \left(\frac{P - c_i - w_i}{2}\right) dc_i = A - P,$$

where $N = \{c_i \in [0,1]/$ firm i is not linked$\}$ and $L = \{c_i \in [0,1]/$ firm i is linked$\}$.

This leads to the following equilibrium price:

$$P^*(a, b, w) = \frac{2A + l(1-a) + n + b(1-z) + wz}{3}, \tag{2.1}$$

where z is the mass of set N (hence, the mass of set L amounts to $1 - z$), $n = \int_N c_i dc_i$ and $l = \int_L c_i dc_i$.

We focus on an interior equilibrium in which all downstream firms produce at this price. This is the case if $b,\ w \leq \dfrac{2A}{3} - 1$, $0 \leq a \leq 1$ and $A > 6$.

In the second stage, and given the set of linked firms, the monopolist chooses the personalized wholesale prices and the posted price to maximize:

$$\Pi^I(L, a, b, w) = \int_N w \left(\frac{P^*(a,b,w) - c_i - w}{2}\right) dc_i + \int_L w_i \left(\frac{P^*(a,b,w) - c_i - w_i}{2}\right) dc_i,$$

where superscript I denotes Game I.

The optimal contracts are given by[3]:

$$a^* = \frac{1}{2},\ b^* = \frac{A}{2} \text{ and } w^*(L) = \frac{A}{2} - \frac{n}{2z}. \tag{2.2}$$

Notice that, due to the linearity of the model, the personalized contract does not depend on the distribution of links. Plugging (2.2) into (2.1) we get the equilibrium price:

$$P^*(a^*, b^*, w^*) = \frac{1 + 10A}{12}.$$

[3]Second order conditions are dealt with in Appendix A.

Observe that the equilibrium price does not depend on the distribution of links either[4].

The equilibrium profits of the upstream firm amounts to:

$$\Pi^I(L, a^*, b^*, w^*(L)) = \frac{3n^2 + (2A(A-1) + 3\widehat{l} - \frac{1}{4})z}{24z}, \tag{2.3}$$

where $\widehat{l} = \int_L c_i^2 dc_i$.

In the first stage, we have to derive the equilibrium distribution of links depending on whether the links are created by the upstream firm or by downstream firms. We analyze each case in turn.

2.1. The upstream firm chooses the links.

We proceed to characterize the first stage equilibrium of Game I. The decisions of the upstream firm are (1) to choose the number of unconnected firms (that amounts to decide the mass z of set N) and (2) to decide the distribution of connected and unconnected firms (that amounts to the choice of terms n and $\widehat{l}$ in the profit expression).

In the following lemma, we find the distribution of connected firms that maximizes (2.3), for a given measure $1-z$ of linked firms. Observe that this amounts to maximize $n^2 + \widehat{l}z$.

Lemma 2.1. *In Game I, for a given measure* $1-z$ *of linked firms, their optimal distribution for the upstream firm is any set* $B = [0, s] \cup [z+s, 1]$ *for any* $s \in [0, 1-z]$.

Proof. For the distributions not considered in the lemma, there exist numbers $0 \leq a < b < d \leq 1$ and $0 < c < d - b$ such that firms in $[a, b] \cup [b+c, d]$ are not connected and the ones in $[b, b+c]$ are connected. If z is the measure of unconnected firms and H is the expected cost of unnconected firms other than the ones in $[a, b] \cup [b+c, d]$, the payoff of the upstream

[4] Yoshida (2000) considers a more general transformation technology for the inputs. For the particular case of one-to-one relationship between input and output, however, the equilibrium price is the same with and without price discrimination. We prove that this results extends to any possible level of partial price discrimination.

firm can be written as:

$$f(b)=\left(H+\int_a^b c_i dc_i+\int_{b+c}^d c_i dc_i\right)^2+z\int_b^{b+c} c_i^2 dc_i+constant$$

We show that these distributions are not optimal because the payoff can be increased changing b, given that it is strictly convex in b.

$$f^{'}(b)=2\left(H+\int_a^b c_i dc_i+\int_{b+c}^d c_i dc_i\right)(b-(b+c))+z\left((b+c)^2-b^2\right)$$

which can be rewritten as:

$$f'(b)=-2c\left(H+\int_a^b c_i dc_i+\int_{b+c}^d c_i dc_i\right)+(2b+c)cz$$

The second derivative is given by:

$$f"(b)=2c^2+2cz>0$$

The convexity of the payoff function implies that these distributions do not maximize the profits of the upstream firm. If we allow the upstream to choose parameter b, it will choose either $b=a$ or $b=d-c$ increasing its profits. In any case, the set $[b,b+c]$ will not be any longer an "island" of linked firms. Iterating this process, we will end up with a distribution of linked firms included in set B in the lemma and, given the convexity of the payoff function, the payoff of the upstream firm increases. Now, in order to check that any distribution in the lemma is optimal, observe that he payoff of the upstream firm with any these distributions is:

$$h(s)=\left(\int_s^{z+s} c_i dc_i\right)^2+\left(\int_0^s c_i^2 dc_i+\int_{z+s}^1 c_i^2 dc_i\right)z$$

We obtain the first derivate:

$$h'(s)=2\left(\int_s^{z+s} c_i dc_i\right)(z+s-s)+\left(s^2-(z+s)^2\right)z$$

Rearranging terms we obtain that the function is constant in s:

$$h'(s) = \left((z+s)^2 - s^2\right) z + \left(s^2 - (z+s)^2\right) z = 0$$

Therefore, all the distributions in the lemma yield the same payoff. ■

In order to understand the intuition behind the above result observe that, if all firms were linked, every firm would be charged a personalized price equal to $w_i = \frac{A}{2} - \frac{c_i}{2}$. Given that connecting firms is costly, a subset of firms will not be linked and they will be charged a price that is an average of their personalized prices. The gain of price discrimination is higher the higher the difference between the personalized prices and the posted price, which is achieved by linking the firms with more "extreme" cost parameters.

Let us consider again the original game. The following lemma relates the equilibrium payoff of the upstream firm in the second stage of both games.

Lemma 2.2. *If $w^*(L) \geq b^* - a^* c_i$ for all $c_i \in L$, then (2.2) and (2.3) are the equilibrium contracts and payoff of the upstream firm in the second stage of the original game. Otherwise, the equilibrium payoff in the second stage of the original game is strictly lower than (2.3).*

Proof. In Game I, let us define the maximal value of $\Pi^I\ (L, a, b, w)$ as $\Pi^{I*}(L)$.

In the original game, let us define the payoff of the upstream firm given the contracts (a, b, w) as $\Pi(L,\ a, b, w)$. Assume this function is maximized in (a', b', w'). Let us define $Z = \{c_i \in L\ /\ c_i \geq \frac{b' - w'}{a'}\}$. It is possible to see that:

$$\Pi(L, a', b', w') = \Pi^I(Z, a', b', w') \leq \Pi^{I*}(Z) \leq \Pi^{I*}(L)$$

The first equality is driven by the fact that in both cases we have the same set of linked firms and the contracts are also the same. In this case, we will have the same equilibrium market price and the same payoffs.

If $Z = L$ and $(a', b', w') = (a^*,\ b^*,\ w^*(L))$, the two inequalities transform into equalities.

If $Z = L$, and $(a', b', w') \neq (a^*,\ b^*,\ w^*(L))$, the first inequality is strict.

If $Z \subset L$ the second inequality is strict and comes from the fact that the equilibrium payoffs in Game I increase by adding links. To prove that, assume that firms in the interval $[a, a+t]$ are connected, firms in $[a+t, c]$ are unconnected, H is the expected costs of unconnected firms other that the ones in $[a+t, c]$ and z its measure. Following (2.3), the equilibrium payoffs are given by:

$$f(t) = \frac{1}{8}\left(\frac{\left(H + \int_{a+t}^{c} c_i dc_i\right)^2}{z + c - a - t} + \int_a^{a+t} c_i^2 dc_i + K\right)$$

where K is a constant

It is immediate to see that $f'(t) > 0$, which proves the second inequality. ■

If the upstream firm connects the downstream firms in $J = [z, 1]$, it will set in Game I the contracts in (2.2), where $w^*(J) = \frac{A}{2} - \frac{z}{4}$. But as we have that the highest personalized contract in J is lower than $w^*(J)$ (observe that $\frac{A-z}{2} < \frac{A}{2} - \frac{z}{4}$), these will also be the equilibrium contracts in the original game. Therefore, the upstream firm will obtain the payoffs given in (2.3). We are going to show, with the help of Lemmas 2.1 and 2.2, that this is the optimal distribution of links given that the upstream firm connects a mass $1 - z$ of firms.

Observe that any set B in Lemma 2.1 (other than $J = [z, 1]$), has an interval $[0, s]$, with $s > 0$, that includes the firm with the lowest cost. For this firm (and for some neighborhood of firms with strictly positive mass around 0), the personalized price $w_i(c_i = 0) = \frac{A}{2}$ would be strictly higher than $w^* = \frac{A}{2} - \frac{n}{2z}$; hence, these firms would refuse the personalized price and would attend instead the posted price market. Given Lemma 2.2, the upstream firm would obtain, with these distribution of links, less profits than in Game I and, therefore, less profits than if he connects the firms in $[z, 1]$. With distributions of links different than the ones in Lemma 2.1, the profits of the upstream firm are bounded above by the profits he obtained in Game I and, therefore, they are lower than the ones obtained connecting firms in $[z, 1]$.

Transforming (2.3) taking into account that only firms in $[z, 1]$ are connected, we can obtain the payoff of the upstream firm in the first stage as a function of z, which is given by:

$$\frac{-\frac{1}{4}z^3 + (2A(A-1) + \frac{3}{4})}{24} - (1-z)f. \tag{2.4}$$

We are now ready to establish the main result of this section.

Proposition 2.3. *The upstream firms connects the firms in the set* $[Min\{z^u(f), 1\}, 1]$ *where* $z^u(f) = 4\sqrt{2f}$.

Observe that the size of demand (A) does not affect the set of linked firms. Of course, it affects upstream gains and losses of connecting downstream firms, but in the aggregate these effects cancel out. Notice that creating a link has a direct positive effect of allowing the upstream firm to personalize the contract, which is indeed increasing in A. On the other hand, it has an indirect effect because the upstream firm will adjust the posted price upwards. It turns out that this indirect effect is decreasing in A and it exactly cancels out with the direct effect.

2.2. Downstream firms choose the links.

First of all, we show that an equilibrium with connected firms is defined by a cut-off value z such that all firms in $[z, 1]$ get connected.

Assume a candidate equilibrium set L of linked firms. Denote by a^+, b^+ and w^+ the equilibrium contracts given this distribution of links. Then, it must be the case that a downstream firm with cost c_i is connected in equilibrium if and only if:

$$F(a^+, b^+, w^+) = \left(\frac{P^*(a^+, b^+, w^+) - w^+ - c_i}{2}\right)^2 - \left(\frac{P^*(a^+, b^+, w^+) - b^+ - (1-a^+)c_i}{2}\right)^2 + f \leq 0. \tag{2.5}$$

This requires that for all $c_i \in L$, $w^+ > b^+ - a^+ c_i$.

In other words, a firm i is connected in equilibrium only if its personalized price is lower than the posted price. But then, the maximization program of the upstream firm is such that the restriction that linked firms only use the personalized contract if it offers better terms that the posted market is not binding. This implies that the optimal contracts must be the same as the ones in Game I. They are given by (see 2.2) :

$$a^{+} = \frac{1}{2},\ b^{+} = \frac{A}{2} \text{ and } w^{+} = \frac{A}{2} - \frac{n}{2z}.$$

Plugging the optimal contracts into expression (2.5), we obtain:

$$F(\frac{1}{2},\frac{A}{2},\frac{A}{2}-\frac{n}{2z}) = \left(\frac{\frac{10A+1}{12} - \left(\frac{A}{2}-\frac{n}{2z}\right) - c_i}{2}\right)^2 - \left(\frac{\frac{10A+1}{12} - A/2 - c_i/2}{2}\right)^2 + F.$$

We now check that this function is strictly decreasing in c_i:

$$\frac{\partial F}{\partial c_i} = (\frac{1}{8})(-A + 3c_i - \frac{10A+1}{6} + 4\left(\frac{A}{2}-\frac{n}{2z}\right)) < (\frac{1}{48})(-1 - 4A + 18c_i) < 0 \text{ if } A > 5.$$

This implies that if (2.5) is satisfied for a firm with cost parameter c_i, it must also hold for less efficient firms. In other words, the equilibrium must have a cutoff structure, where only high-cost firms decide to establish a link.

Next, we have to look for the equilibrium value of z. If firms with $c_i \in [z, 1]$ are connected, the optimal posted price in Game I is $\frac{A}{2} - \frac{z}{4}$ and it is higher than the personalized price received by any linked firm:

$$\frac{A}{2} - \frac{z}{4} > \frac{A-z}{2} \geq \frac{A-c_i}{2}, \text{ for all } c_i \in [z, 1].$$

Then, by using Lemma 2.2, we know that the original game has the same equilibrium contracts as Game I.

Therefore, in order to calculate the equilibrium value of z we have to solve:

$$F(\frac{1}{2},\frac{A}{2},\frac{A}{2}-\frac{z}{4}) = \left(\frac{1}{192}\right) z(-2 - 8A + 15z) = 0.$$

It has only one solution in $[0,1]$, denoted by z^*. Then, the equilibrium distribution of links is given in the following proposition.

Proposition 2.4. *The equilibrium set of connected firms is given by* $[\min\{z^d(f),1\},1]$, *where* $z^d(f) = \dfrac{4A+1-\sqrt{(4A+1)^2-2880f}}{15}$.

Observe that when $f \geq \dfrac{8A-13}{192} = f^{\max}$, no firm creates a link in equilibrium.

We have that the number of links created is decreasing in f and increasing in A. Comparing Propositions 2.3 and 2.4, we can conclude that more links are created when downstream firms create the links than when the upstream firm creates the links. In the former case, the firms that invest in the link impose a negative externality on the firms that remain in the posted price market, in the form of a higher wholesale price. This leads downstream firms to create too many links. In fact, if they could decide the number of links in order to maximize joint downstream profits, they would create no link at all. In the latter case, the upstream firm takes into account the externality when deciding the number of links to be created. As we have seen, as a result, it creates less links.

Regarding social welfare, it is crucial to notice that total output does not depend on the distribution of links. Then (partial) price discrimination only affects costs. We know that the upstream firm through price discrimination subsidizes inefficient firms what increases total costs. Then welfare will be maximized when price discrimination is forbidden. the third stage equilibrium price does not depend on x^S. Therefore, in order to maximize social welfare it is enough to minimize total costs (production and linking costs). Production costs will be minimized when marginal costs, wholesale price excluded ($c_i + 2q_i$), are the same in equilibrium for all downstream firms. This can be the case only when all downstream firms receive the same wholesale price i.e. when all firms attend the posted price market. Then he would set a prohibitively large tax so that no firm gets connected.

3. The case of endogenous cost.

Up to now, the cost of a link has been assumed to be exogenous. It seems interesting to analyze the case where this cost becomes endogenous. One possibility is that there exists an intermediary in charge of connecting the upstream and downstream firms. In this case the cost of the link would be represented by a subscription fee that the intermediary would charge both to the upstream and any downstream firm willing to establish a personal relationship. We assume that the creation of a link between the upstream firm and downstream firm i occurs only when both of them pay the subscription fee.

Let us analyze the following game. First, the intermediary chooses the subscription fee for the upstream firm (s_u) and for downstream firms (s_d). Second, the upstream firm chooses for which downstream firms to pay the subscription fee. Third, each downstream firm decides whether or not to pay the subscription fee. In the fourth stage, the upstream firm sets the supply contracts. Finally, market competition takes place. We solve the game by backward induction.

Stages four and five are like in the previous model. Observe that, for simplicity and in order to avoid coordination failures, we assume that the upstream firm pays the subscription fees before downstream firms. The solution of these two stages is a direct application of the results of the previous sections. Assume that the upstream firm has paid the subscription fee for downstream firms in the set $[z, 1]$. This is the relevant case, because we know that the upstream firm wants to connect inefficient firms. In the third stage, in order to determine which downstream firms will pay the fee, it is useful to recall what would happen in the model where links were decided by downstream firms and $f = s_d$: in this case, firms in the set $[z^d(s_d), 1]$ would be connected. But now we have the additional restriction that the upstream firm has to have paid the subscription fee. Then, the set of downstream firms that will pay the fee in equilibrium is given by $[Max\{z^d(s_d), z\}, 1]$.

In order to solve the second stage, we have to recall the result when the upstream chose the links and $f = s_u$. It connected the firms in the set $[z^u(s_u), 1]$. But now, the upstream firm has to be sure that the corresponding downstream firms will pay the fee. Then, in equilibrium it will pay the fee for firms in $[Max\{z^u(s_u), z^d(s_d)\}, 1]$.

In the first stage, the objective of the intermediary firm is given by:

$$(s_u + s_d)\left(1 - Max\{z^u(s_u), z^d(s_d)\}\right).$$

It is easy to see that, in equilibrium, it must be the case that s_u and s_d are chosen so that $z^u(s_u) = z^d(s_d)$. The reason is the following: imagine that $z^u(s_u) < z^d(s_d)$. In this case, the upstream firm could increase profits by increasing slightly s_u. Observe that this would not change the number of links created but it would allow the intermediary to obtain more revenues from the upstream firm. Finally, we solve the maximization program of the intermediary making use of $z^u(s_u) = z^d(s_d)$, and get the equilibrium subscription fees s_u^* and s_d^* (the actual values are complex and not very informative an they can be seen in Appendix B). We check that they are positive and increasing in the size of demand (A). However, s_d^* is much smaller and more sensitive than s_u^* to changes in A. In particular, $\underset{A->\infty}{Lim} s_d^* = \infty$ and $\underset{A->\infty}{Lim} s_u^* = \frac{1}{128}$. The reason of the low upper bound for s_u^* is that the links that the upstream firm is willing to create do not depend on A and it creates no link whenever $s_u \geq \frac{1}{32}$. As the intermediary firm finds profitable to create links it will set the subscription fee at a lower level than this limit. Observe that the profitable side of the market is the downstream sector where the subscription fee grows without bounds. In fact, even the existence of a small transaction cost (k) of collecting the fees of the upstream firm would induce the intermediary firm to provide the connection to the upstream firm for free. A sufficient condition for this to hold is that $k \geq \frac{1}{128}$, which is the upper bound of the revenues obtained from the upstream firm in the model.

Summarizing, we obtain partial price discrimination as the equilibrium outcome of an extended game where the cost of creating links is endogenously created by an intermediary firm. We obtain as before that high cost firms are the ones treated personally. Moreover, we obtain that the burden of the (endogenous) cost of price discrimination mainly falls on the downstream side of the market.

4. Discussion and conclusion.

The literature on third-degree price discrimination has mainly focused on its effect on social welfare. Profitability of price discrimination was taken for granted given that no cost was associated to it. Our contribution to the literature is to recognize that customizing prices may be costly, which creates an interesting trade-off. The upstream firm, before deciding whether to personalize the price of a customer, has to balance its possible gains with its cost. As a result, he decides to pay the cost for his most valuable customers and charge a uniform price to the rest of firms. We name this situation as partial price discrimination and encompasses as particular cases those of uniform price and complete price discrimination that have focused the attention of the literature so far.

We have considered a model with an upstream monopolist selling an input to a continuum of heterogeneous downstream firms producing an homogeneous good. We show that, regardless of whether the cost of the links is paid by the upstream firm or downstream firms, inefficient firms receive personalized prices. Furthermore, when the cost of the link is endogenized through an intermediary, we obtain that most of the intermediation profits are obtained from the competitive downstream sector. One is left to wonder whether the results crucially depend on the structure of the upstream and downstream sector. It is straightforward to show that the same results can be obtained if we assume instead that the upstream sector is competitive and we have a monopoly downstream. In this case, however, most of the profits of intermediation are paid by the (competitive) upstream sector. This indicates that the side of the market that bears the cost of intermediation depends on the horizontal level of competition and not on whether it is a buyer or a seller in the input market.

For simplicity we have analyzed the case where the upstream firm perfectly distinguishes the cost of every single firm; that is we assume a complete information framework. It would be interesting to analyze the polar case where the cost is private information of each firm. This would limit the ability of the upstream firm to price discriminate and would change dramatically the structure of the model. The analysis is carried over in the companion paper Bru et al. (2005).

5. References

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6. Appendix A

Second order conditions of the maximization program of the upstream firm are satisfied if:

$$F = \left(\int_L c_i dc_i\right)^2 - \int_L dc_i \int_L c_i^2 dc_i < 0$$

for any distribution of links. Then,

$$\text{If } L = [a,b], \text{ then } F = -(\frac{1}{12})(b-a)^4 < 0. \tag{6.1}$$

Suppose now that $L = T \cup [a,d]$, where T has a positive measure. Then

$$\begin{aligned}
F &= \left(\int_T c_i dci + \frac{d^2-a^2}{2}\right)^2 - \left(\int_T dci + d - a\right)\left(\int_T c_i^2 dci + \frac{d^3-a^3}{3}\right) \\
\frac{\partial F}{\partial d} &= 2\left(\int_T c_i dci + \frac{d^2-a^2}{2}\right)d - \left(\int_T dci + d - a\right)d^2 - \left(\int_T c_i^2 dci + \frac{d^3-a^3}{3}\right) \\
&= -\int_T (d^2 + c_i^2 - 2dc_i)dc_i - a^2 d + ad^2 - \frac{d^3-a^3}{3} \\
&= -\int_T (d-c_i)^2 dc_i + f(d) < 0
\end{aligned}$$

$$\begin{aligned}
f(d) &= -a^2 d + ad^2 - \frac{d^3-a^3}{3} \\
f'(d) &= -a^2 - d^2 + 2da = -(a-d)^2 < 0 \\
\text{As } f(a) &= 0,\ f(d) \leq 0 \text{ for } d \geq a
\end{aligned}$$

Then, in order to sign F for any distribution of links we proceed the following way. If we disconnect all segments but one, F (for 6.1) is negative. The result above proves that connecting the other segment has a negative effect on F. Therefore $F < 0$ for all distribution of links.

7. Appendix B: The equilibrium subscription fees.

$$
\begin{aligned}
s_d^* &= \frac{-371 - 32(-4+A)A + 2(23+2A)\sqrt{(67+8A(-5+8A)}}{23328} \\
s_u^* &= \frac{94 + 68A + 64A^2 - (11+8A)\sqrt{67+8A(-5+8A)}}{11664}
\end{aligned}
$$