Production Functions with imperfect competition*

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Abstract

In this paper I address two problems in the production function estimation literature. The first problem is input endogeneity and the second is the use of deflated sales as a proxy for output when there is imperfect competition. Using a demand system and allowing input demand to depend on the individual state variables as well as on the industry equilibrium I explain how to jointly recover the production function parameters and demand elasticity.

Keywords: Productivity, Production Function, Differentiated Products

1 Introduction

In this section I analyze the effect of specifying a fully dynamic industry investment on estimating the production function. The main problem that arises in an imperfect competition setting is that demand elasticity can no longer be recovered in the first stage as proposed by Levinsohn and Melitz (2005) and De Loecker (2007). This is due to the fact that input demand (either investment or materials) are functions of aggregate market conditions. I present evidence of the biased demand elasticity estimates.

2 Estimating Production Functions

The traditional approach to estimating production functions dates back is to Cobb and Douglas (1928) and some of its problems have been detected since Marschak and Andrews (1944). Currently there have been some attempts to solve the input endogeneity problem either via controlling for productivity (Olley and Pakes, 1995, henceforth O&P; Levinsohn and

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Petrin, 2003, henceforth L&P) or via introduction of adjustment costs (Bond and Soderbom, 2005, henceforth B&S). A second problem has been the use of revenues instead physical output when markets are not perfectly competitive (Klette and Griliches, 1996). Recently De Loecker (2007) and Levinshon and Melitz (2005) have proposed a framework which accounts for the two problems jointly. In this paper I show that the methodology is inconsistent with a industry dynamic equilibrium framework similar to Ericson and Pakes (1996). The main problem is that the demand elasticity cannot be recovered in the first step. I propose a way to deal with this problem by recovering demand elasticity in the second step.

Finally Buettner (2005) and Doraszelski and Jaumandreu (2007) propose alternative ways to relax the exogenous Markov process for productivity by allowing this to be controlled by R&D expenditures. In my case I allow productivity to follow a controlled Markov process of a special form which depends only on whether firms are R&D performers or not.

2.1 Demand

Using the Dixit-Stiglitz monopolistic competition framework demand can be written as:

$$Q_i = \tilde{Y}\tilde{P}^{\sigma-1}P_i^{-\sigma} \tag{1}$$

Where $\left(\frac{\tilde{Y}}{\tilde{P}}\right) = \frac{\sum_{i=1}^{N_t} P_i Q_i}{\tilde{P}}$ is total industry deflated revenues.

2.2 Production function

The production technology uses both capital (K) and labor (L) with a given productivity factor (ω) according to a Cobb Douglas

$$Q_i = \omega_i L_i^{\alpha} K_i^{\beta} \tag{2}$$

2.3 Productivity

Productivity is not directly observed but there are methods[1] to estimate it as the residual from a production function estimation (Olley and Pakes, 1995; Levinshon and Petrin, 2003; De Loecker, 2007). To be coherent with the theoretical model I use a methodology similar to De Loecker (2007) which allows me to recover both the production function parameters and the demand elasticity when one uses deflated sales instead of quantities. The main problem with De Loecker (2007) is that it is inconsistent with the model I have outlined. The reason for the inconsistency arises from the fact that input demand function depend on the industry state, more precisely on the aggregate industry state. This means that the

[1] Ackerberg et al. (forthcoming) provide a survey on the literature for estimating production functions.

elasticity of demand cannot be recovered in the first step since the input demand is also a function of the aggregate sales and I can only recover it only in the second step together with the capital coefficient. To see this notice that sales are $P.Q$ so taking the logs and using (1) and (2) from above:

$$y_{it} = p_{it} + q_{it} = \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\tilde{p}_t + \frac{\sigma-1}{\sigma}\left(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}\right) + \varepsilon_{it}$$

or

$$y_{it} - \tilde{p}_t = \frac{1}{\sigma}(\tilde{y}_t - \tilde{p}_t) + \frac{\sigma-1}{\sigma}\left(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}\right) + \varepsilon_{it}$$

The first problem is then that the unobserved productivity term ω_{it} is possibly correlated with the inputs. Levinshon and Petrin (2003) propose the use of materials to control for the unobservable. To see this recall that input demand is a function of productivity

$$m_{it} = m(\omega_{it}, k_{it}, R_{it}, \tilde{y}_t)$$

Assuming invertibility this can be expressed as[2]

$$\omega_{it} = \omega(k_{it}, R_{it}, \tilde{y}_t, m_{it})$$

and the unobservable is now a function of observables. Note however that since productivity is also a function of market conditions ($\tilde{y}_t$), demand elasticity (σ) cannot be recovered in the first stage.

Imposing that productivity is governed by a controlled first order Markov process we get

$$\omega_{it} = E[\omega_{it}|\omega_{it-1}, R_{it-1}] + \nu_{it}$$

2.3.1 First stage

From above we can rewrite the production function as (using deflated sales as variables $y^p_{it} = y_{it} - \tilde{p}_t$)

$$\begin{aligned} y^p_{it} &= \frac{1}{\sigma}\tilde{y}^p_t + \frac{\sigma-1}{\sigma}\left(\alpha_k k_{it} + \alpha_l l_{it}\right) + \frac{\sigma-1}{\sigma}\omega_{it} + \varepsilon_{it} \\ &= \frac{\sigma-1}{\sigma}\alpha_l l_{it} + \phi(k_{it}, R_{it}, \tilde{y}^p_t, m_{it}) + \varepsilon_{it} \end{aligned}$$

[2] A slight concern with invertibility and imperfect competition is the fact that with imperfect competition an increase in productivity might not lead to a direct increase in output and therefore in materials usage. For the demand system specified, an increase in productivity is equivalent to a decrease in costs and it translates directly into a decrease in prices (equation **??** in appendix). This means total output goes up and therefore also does materials usage.

where

$$\phi(k_{it}, R_{it}, \tilde{y}_t^p, m_{it}) = \frac{1}{\sigma}\tilde{y}_t^p + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + \frac{\sigma-1}{\sigma}\omega(k_{it}, R_{it}, \tilde{y}^p, m_{it})$$

And we can estimate this non-parametrically using an nth-order polynomial. This provides estimates of $\widehat{\frac{\sigma-1}{\sigma}\alpha_l}$ and $\hat{\phi}$.

2.3.2 Second stage

For the second stage I use the estimated values to construct

$$\hat{\phi}_{it} = \hat{y}_{it} - \widehat{\frac{\sigma-1}{\sigma}\alpha_l} l_{it}$$

with this we can give an estimate of $\frac{\sigma-1}{\sigma}\omega_{it}$ for a given $\widetilde{\frac{\sigma-1}{\sigma}\alpha_k}$ and $\widetilde{\frac{1}{\sigma}}$

$$\widehat{\frac{\sigma-1}{\sigma}}\omega_{it} = \hat{\phi}_{it} - \widetilde{\frac{1}{\sigma}}\tilde{y}_t + \widetilde{\frac{\sigma-1}{\sigma}}\alpha_k k_{it}$$

Using this we can approximate non-parametrically $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$ with an nth-order polynomial

$$\begin{aligned}
y_{it} - \widehat{\frac{\sigma-1}{\sigma}}\alpha_l l_{it} &= \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + E[\omega_{it}|\omega_{it-1}, R_{it-1}] + \nu_{it} + \varepsilon_{it} \qquad (3)\\
&= \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + \\
&\quad + \left[\begin{array}{c} \gamma_0^0 + \gamma_1^0\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right) \\ +\ldots + \gamma_n^0\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right)^n \end{array}\right] 1\left[R_{it-1} = 0\right] \\
&\quad + \left[\begin{array}{c} \gamma_0^1 + \gamma_1^1\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right) \\ +\ldots + \gamma_n^1\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right)^n \end{array}\right] 1\left[R_{it-1} = 1\right] \\
&\quad + \nu_{it} + \varepsilon_{it}
\end{aligned}$$

Using non-linear least squares allows us to finally recover an estimate for $\frac{1}{\sigma}$ and $\frac{\sigma-1}{\sigma}\alpha_k$.

Potential problems in the second stage For the second stage estimation to work, the error term of equation (3), $\nu_{it} + \varepsilon_{it}$, must be uncorrelated with k_{it} and $\tilde{y}_t$. While this might be a reasonable assumption for k_{it} due to the timing of investment that makes k_{it} independent from 'news' in period t, the same is not necessarily true for $\tilde{y}_t$ if in the productivity shock ν_{it} there is an aggregate time component ν_t not captured by $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$. One potential instrument is the use of lagged $\tilde{y}_{t-1}$.

I also acknowledge the criticism by Ackerberg, Caves and Frazer (2006) on the potential multicollinearity problem between l_{it} and $(k_{it}, R_{it}, \tilde{y}_t, m_{it})$. However, the perfect collinearity does not arise in the data. This might actually not be severe if all we want is to recover an estimate for productivity and not for the production function coefficients but might raise concerns about misspecification that I do not address in this paper and leave for future research.

A further problem is the sample selection due to exit. As explained by Olley and Pakes (1995), this selection problem arises if big firms are more likely to exit upon a negative shock which generates negative correlation between productivity and capital stock for the firms who remain in the industry. However, this fact is likely to be relevant in industries with severe exit behavior, but it is unlikely that this is true for industries with little exit. For this reason I will not correct the estimates for sample selection.

2.4 Adjustment cost literature

Somewhat related the adjustment cost literature has evolved using advanced dynamic panel data specifications. Bond and Soderbom (2005) propose an adjustment cost model that can solve the multicollinearity problems between labor and materials has also explained in Ackerberg, Caves and Frazer (2006)[3]. This way assuming productivity follows a particular Markov transition, first-order autoregressive. Since they do not specify R&D into their model, I just assume two different AR(1) processes for R&D and non-R&D firms

$$\omega_{it} = \begin{cases} \rho^0 \omega_{i,t-1} + \nu_{it} & \text{if } R_{i,t-1} = 0 \\ \rho^1 \omega_{i,t-1} + \nu_{it} & \text{if } R_{i,t-1} = 1 \end{cases}$$

This way we can specify the production function as

$$y_{it} - \tilde{p}_t = \frac{1}{\sigma}(\tilde{y}_t - \tilde{p}_t) + \frac{\sigma - 1}{\sigma}(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}) + \varepsilon_{it}$$

quasi-differencing we get (where subscript p denotes deflated values and subscript j denotes $R_{i,t-1} = 0$ or not)

$$y^p_{it} - \rho^j y^p_{i,t-1} = \frac{1}{\sigma}\left(\tilde{y}^p_t - \rho^j \tilde{y}^p_{t-1}\right) + \frac{\sigma - 1}{\sigma}\left(\omega_{it} - \rho^j \omega_{i,t-1} + \alpha_k\left(k_{it} - \rho^j k_{i,t-1}\right) + \alpha_l\left(l_{it} - \rho^j l_{i,t-1}\right)\right) + \varepsilon_{it} - \rho^j \varepsilon_{i,t-1}$$

Or

$$y^p_{it} - \rho^j y^p_{i,t-1} = \frac{1}{\sigma}\left(\tilde{y}^p_t - \rho^j \tilde{y}^p_{t-1}\right) + \frac{\sigma - 1}{\sigma}\left(\alpha_k\left(k_{it} - \rho^j k_{i,t-1}\right) + \alpha_l\left(l_{it} - \rho^j l_{i,t-1}\right)\right) + \frac{\sigma - 1}{\sigma}\nu_{it} + \varepsilon_{it} - \rho^j \varepsilon_{i,t-1} \tag{4}$$

[3] Doraszelski and Jaumadreu propose the use a parametric input demand specification that arises naturally for the Cobb-Douglas production function case to solve this problem.

which I estimate using a dynamic panel data technique. I estimate this in two stages. In the first stage I estimate the full equation without imposing the constraint on the lagged variables for $k_{it-1}, l_{it-1}, \tilde{y}_{t-1}$ estimating

$$y_{it}^{p} = \pi_0 y_{i,t-1}^{p} + \pi_1 \tilde{y}_t^{p} + \pi_2 \tilde{y}_{t-1}^{p} + \pi_3 k_{it} + \pi_4 k_{i,t-1} + \pi_5 l_{it} + \pi_6 l_{i,t-1} + \tilde{\nu}_{it}$$

I recover an estimate for $\hat{\rho} = \hat{\pi}_0$ and reestimate the model imposing the constraints on the parameters. I run this separately for R&D and non-R&D firms.

3 Results

In this section I compare the results for the alternative methodologies using data for the Portuguese Moulds Industry over the period 1994-2003.

Table 1 contains the results for a simple OLS and fixed effects specification. In Table 2 I estimate the original O&P model using investment to control for productivity, using the original specification in column (i), and allowing for imperfect competition recovering demand elasticity in the first stage (column (ii)) or in the second stage (column (iii)). In Table 3 I reestimate the model using materials input to control for productivity as proposed by Levinsohn and Petrin. As for the O&P specification, column (i) assumes perfect competition, column (ii) estimates demand elasticity in the first stage and column (iii) allows materials demand to be a function of the aggregate state and recovers demand elasticity in the second stage. In Table 4 I estimate the dynamic production function model as proposed by Bond and Soderbom. Finally Table 5 provides a comparison for the different specifications.

The results in columns (vii) and (xi) of table 5 confirm the bias in the estimates if demand elasticity is recovered in the first stage. The sign of the bias is a priory undetermined, however a negative bias is consistent with a negative correlation between the aggregate demand shock and productivity (or positive correlation between aggregate shock and prices). Notice also the bias in labor and capital coefficients of both O&P and L&P in columns (v) and (ix) when time dummies are not used and imperfect competition effects are not controled for.

I denote a preference for the L&P approach over O&P because of the labor coefficient bias in the first stage if the conditions for investment invertibility fail and productivity is not well controlled. This could be the cause of the upward bias in the labor coefficient with the O&P approach. Curiously, the Fixed Effect results with time dummies in column (iv) are very close to the preferred specification in column (xii) and seems to perform pretty well.

The results using Bond and Soderbom show that the labor and capital coefficients are well estimated but not demand elasticity. This could be due to aggregate shocks being negatively correlated with productivity. Splitting the sample into R&D and non-R&D firms in columns (xiv) and (xv) seems to suggest a higher mark-up for the R&D firms but this issue needs further research.

Note that the potential problem of multicollinearity using L&P as pointed out by Ackerberg et al (2006) and Bond and Soderbom (2005) does not seem to be a major concern since the labor coefficients recovered in the first stage are not significantly different from the ones using B&S.

4 Conclusion

In this paper I have addressed to common problems in the production function literature. The first is very well know and relates to input endogeneity and has been widely studied in the literature. The second is the problem of estimating production functions when competition is imperfect. Even though the problem has been addressed by Levinsohn and Melitz (2005) and De Loecker (2007), both have done this assuming a single agent setting. If one expands this to a dynamic industry model, input demand will be a function of market conditions and demand elasticity can only be recovered in the second stage. I presented evidence that supports the bias in demand elasticity. One curious result is the good performance of a simple fixed effects specification with time dummies. I also register a preference for the Levinshon and Petrin approach as compared to Olley and Pakes. This is due to the potential problems with investment inversion.

References

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