

Sunk Costs of R&D, Trade and Productivity: the moulds industry case*

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Abstract

Evidence suggests that industry productivity improves with international trade via (i) selection of the best firms and (ii) within firm productivity growth. In this article I focus on the second channel operating through innovation. Trade liberalization allows the best firms to exploit the economies of scale in the R&D process and can explain the observed growth. I estimate an equilibrium model of industry dynamics with endogenous productivity. I then show how the European economic integration induced innovation in the Portuguese moulds industry.

Keywords: Industry Dynamics, Innovation, Markov Equilibrium, Moulds Industry, R&D, Structural Estimation, Sunk Costs, Trade

JEL Classification: C61, D92, F12, L11, L22, O31

1 Introduction

Trade liberalization can spin innovation. Firms competing internationally have access to larger markets while also facing tougher competition. Trade liberalization can thus

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increase or reduce the incentives for innovation. In this article I illustrate how trade induced innovation for the case of an export-oriented industry in a small economy. I can abstract from export costs because almost all firms are exporters. With no trade costs, the selection mechanism that has been the focus of the recent literature is not present. Instead, I show how trade created the conditions for the best firms to grow and innovate. The increase in innovation is due to the interaction of heterogeneity and innovation costs. While trade can potentially increase the sales of all the firms, only the best will cross the threshold and pay the costs of innovation.

The recent trade literature emphasizes the importance of firm heterogeneity and trade (e.g. Eaton and Kortum (2002), Melitz (2003), Bernard et al. (2003)). The selection mechanism is in line with the evidence that only a fraction of more productive firms export. Empirical studies have also documented that some firms exhibit productivity growth after they enter the export market (e.g. Pavnick (2002), Bernard et al. (2006), De Loecker (2007)). Some models have then introduced endogenous productivity (e.g. Constatini and Melitz (2008), Aw, Roberts and Xu (2009), Atkenson and Burstein (2010)). Finally, one particular problem with models that use demand functions with constant elasticity of substitution is the neutrality of market structure to market size which is at odds with the empirical results (e.g. Bresnahan and Reiss (1991), Eaton, Kortum and Kramarz (2008)). There has been an attempt to allow for more plausible competition features (e.g. Melitz and Ottaviano (2008), Zhelobodko, et al (2011)).

In this article I incorporate these empirical features. I specify a dynamic equilibrium model with endogenous innovation and physical capital to analyze the effect of trade on equilibrium innovation and R&D. To do so I abstract from the fixed and sunk costs of exporting thereby abstracting from the separate decision of selling to the internal and external markets. In practice this assumption shuts down the selection mechanism, making the interaction of heterogeneity and export costs irrelevant. Instead, I focus on the interaction between heterogeneity and the costs of innovation. Trade liberalization can allow the best firms to exploit the economies of scale and innovate. The framework matches the empirical facts for the Portuguese Moulds industry, where I estimate the model. First, the industry has some characteristics that make it fit to address this particular question. The internal market is negligible and 80% of the firms export 90% of the production. Thus, I can abstract from import competition and focus on the market size effect of trade liberalization. Second, the selection mechanism plays a very marginal

role for this industry allowing me to safely ignore trade costs. Incorporating trade costs would cause the selection mechanism to reinforce the effects analyzed here.

The model is similar in spirit to the industry equilibrium models of Hopenhayn (1992) and Ericson and Pakes (1995). In building the model I bring together three strands from the literature on trade, investment (Cooper and Haltiwanger (2006)) and innovation (Klette and Kortum (2004)). I introduce investment in order to explain the large investment rates observed in this industry. Investment in physical capital is important because firm size is an important explanatory (state) variable for innovation. On the technical side, I introduce incomplete information and assume that individual players do not observe the state or actions of their competitors. This assumption is not implausible for the industry studied. Players' strategies and beliefs are Markovian and depend on payoff relevant states. These assumptions allow me to break the "curse of dimensionality" and solve the model with a large number of players.

Estimation is done in three steps using the method developed by Bajari, Benkard and Levin (2008), henceforth BBL. The first step recovers total factor productivity by estimating a production function. In the second step I estimate the payoff, policy and transition functions. Finally, in the third step I search for the structural (cost) parameters that rationalize the estimated policy functions as optimal. I estimate the model using data for the Portuguese moulds industry. The dataset cover the period 1994 to 2003, after the country joined the EU (1986) and the European Common Market came in place (1993). I then illustrate how the *trade-induced innovation* effect operates by varying market size and simulating the new stationary equilibrium. The results show that a market size reduction to earlier levels is expected to cause a reduction in R&D performance, average productivity, capital stock and number of firms - in line with the evidence. To my knowledge this article is the first to propose and estimate a dynamic equilibrium model with many players using microdata.

The model studied here abstracts from trade costs and is a partial equilibrium analysis. As explained above, abstracting from trade costs is a reasonable approximation for this particular industry and allows me to isolate the mechanism generated by innovation. Extending the model to incorporate trade costs is straightforward and would just require extra data to guarantee identification of the trade costs (i.e. variation in export decisions). Extending the framework to general equilibrium is conceptually possible, but in practice, estimating these models can only be done with aggregate data and very simple settings.

There is thus a difficult trade-off between using rich features in partial equilibrium (the Industrial Organization approach) and estimating more parsimonious general equilibrium models (the Macroeconomics approach).

Given such simplifications, we have to make a careful interpretation of the conclusions that can be drawn. First, when the trade selection mechanism is relevant, we would expect the interaction of innovation and trade to reinforce the effects studied here as illustrated by Aw, Roberts and Xu (2009). On the other hand, as Atkinson and Burstein (2010) show, trade is expected to have general equilibrium economy wide effects on wages that could partially counteract the effects analyzed here.

2 The moulds industry

The Portuguese moulds industry is an interesting case of success. The ability to develop itself even in the absence of a national market and the fact that most of the production has always been exported illustrates the importance of foreign markets. Between 1994 and 2003 the growth in exports was mainly to other European countries in detriment of the US, traditionally the larger market (Table I). After 2001 the industry stabilized and 90% of total production is currently exported (out of which 72% to the automotive industry).

Rank	1970	1980	1985	1990	1995	2000	2003
1	USA	USA	USA	USA	USA	France	Germany
2	UK	UK	UK	France	France	USA	France
3	W. Germ.	Sweden	Russia	Germany	Germany	Germany	Spain
4	Canada	Mexico	Israel	UK	UK	Spain	USA
5	Venez.	W. Germ.	Venez.	Holland	Holland	UK	UK
6	Nd	France	France	Spain	Israel	Sweden	Sweden
7	Nd	Holland	Holland	Sweden	Belg./Lux.	Holland	Holland
8	Nd	Venez.	Sweden	Israel	Sweden	Israel	Romania
9	Nd	Spain	Spain	Belg./Lux.	Brazil	Belg./Lux.	Switzerland

Source: CEFAMOL (2008)

Table I: Ranking of export destinations.

Portuguese mouldmakers benefit from a good international reputation. A report by the US international trade commission (USITC (2002)) emphasizes the fast delivery, technology, quality and competitive price as their main strengths.

"Despite Portugal's small size, it has emerged as one of the world's leading exporters of industrial molds. In 2001, despite limited production of dies,

Portugal was the eighth largest producer of dies and molds in the world and it exports to more than 70 countries. The Portuguese TDM (Tools, Dies and Moulds) industry's success in exporting, and in adoption of the latest computer technologies, has occurred despite the fact that Portugal has a small industrial base on which the TDM industry can depend. Since joining the EU in 1986, Portugal has focused on serving customers in the common market."
 (USITC (2002))

Entering the *European Economic Community* in 1986 and the establishment of the *Common European Market* in 1993 were two important factors that explain the observed growth. With such events, firms were able to reach important European producers and later increase R&D and innovation. Beira et al. (2003) and IAPMEI (2006) document how the close collaboration with car makers constituted a strong push for the development of new processes and products, normally in the form of research projects. These research projects are typically implemented over a given time frame and have well defined targets and objectives. Projects have to be completed, and upon completion the gains become permanent. This type of project-based R&D motivates the sunk cost specification. An alternative current expenditures, or fixed costs, approach would be better suited for industries where instead of projects, R&D is performed continuously (for example the chemical or pharmaceutical industries). Such fixed cost specification would not rationalize the fact that expenditures occur in spans. The data in Tables II and III support these claims. R&D spans last on average 2.5 years and most firms report uninterrupted R&D over the R&D span.¹

		$R_{i,t+1}$	
		0	1
R_{it}	0	93%	7%
	1	45%	55%

Table II: R&D transition probabilities.

Brief industry history

¹Only 16 firms out of 59 interrupt R&D and of these, eight interrupt it for one year, seven for two years and one for five years. The transition probability into R&D is 7.4% and the transition probability out of R&D is 45%.

R&D spans								
Number of years with positive reported R&D	1	2	3	4	5	6	7	8
Number of firms	25	12	6	6	2	5	2	1

Table III: Size of R&D spans.

The industry’s history dates back to the 1930’s and 1940’s when the development of plastics generated a demand for moulds. The Portuguese moulds industry grew in the late 1950’s as a major supplier of moulds to the glass (where it inherited some of its expertise) but mainly to the toy industry. In the late 1980’s the production started shifting from toy manufacturing towards the growing industries of automobiles and packaging. During the 1990’s the largest markets shifted from the US to France, Germany and Spain (Table I, IAPMEI (2006)).

The industry underwent several changes with the introduction of new technologies (*e.g.* CAD, CAM, Complex process, In-mould Assembling) and an increasing importance of innovation and R&D. For example, the technology used in computer operated machines radically changed from the 1980’s to the 1990’s. State of the art machinery allows flexibility at a low cost. It also allows a close collaboration with the client in the design phase. Design teams can work closely with the engineers and produce 3D virtual versions of the mould which is produced in the final phase. Controlling the technology is a major requirement for car-manufacturers.

2.1 Sample description

The sample is part of the *Central de Balanços* compiled by Portuguese Central Bank for the period between 1994 to 2003. It contains detailed financial information and other characterization variables: number of workers, total exports, R&D, founding year and current status. The five-digit industry code (NACE, revision 1.1) is 29563. Aggregate data for total revenues, number of firms, employment and value added are collected from the Portuguese National Statistics Office (INE (2007)) and industry prices from IAPMEI (2006). The Appendix provides a description of the sample and variable construction.

There are 1,290 observations for 231 firms, out of which 265 observations with positive R&D corresponding to 59 firms and 49 R&D start-ups (Table A.II). Average firm has revenues of 1.5 million Euros and employs 32 workers with a labor productivity of 20,381 Euros. The industry is populated by many small and medium firms and no market

leaders. R&D firms are three times larger, 20% more productive and export more (Table A.I).

It is important to notice that the costs of doing R&D are much higher than reported values. The reason being that some costs involve, searching for new technologies, changing the production layout, stop production, training to the employees, etc. Thus, innovation costs cannot be recovered or compared to the reported data. The same applies to physical capital investment.

Stationarity Over the period, production grew at an average 8.9% per year and labor productivity at 5.7% (Tables A.III and A.I) raising some concerns over non-stationarity. Production grows substantially over 1994 to 1998 period and stabilizes thereafter (Figure A.1). The growth and stabilization are consistent with a structural shock occurring somewhere before 1994 with the observed years of 1994 to 1998 being a period of transitional dynamics. This structural shock was the European economic integration that culminated with the establishment of the *Common Market* in 1993. Thus, I abstract from any non-stationarity concerns.

3 The framework

I will start by outlining the model to be estimated and describe the elements of the general case together with a characterization of the equilibrium in the Technical Appendix. The framework captures most of the industry features described above, namely, the high investment rates and the increase in Research and Development. Firms can increase size by investing in physical capital or increase productivity by engaging in R&D activities. R&D and innovation takes the form of projects, implemented over a certain period. Since the projects cannot be undone, the cost of improving productivity is sunk and cannot be recovered. The firms can also enter the market by paying an entry fee and exit the market and collect a scrap value. To close the model we have to define an equilibrium for the dynamic game played by the firms. Since the industry is very fragmented (largest market share is below 10% in any given year), I will abstract from strategic interactions and summarize industry competition with an aggregate state. Firms have rational expectations about future competition. In equilibrium, the expectations (beliefs) must match

actual behavior. Aggregate uncertainty and the discrete number of players leads to a stochastic equilibrium.²

The three features that distinguish this from previous work are the introduction of size (capital), endogenous productivity and aggregate uncertainty. Changes in aggregate uncertainty generate investment sensitivity. Having access to larger markets triggers the size effects induced by endogenous capital choices. Finally, changes in size will influence R&D choices and consequently productivity.

Notice that the unimportant internal market allows me to both ignore import competition and abstract from modeling export decisions separately. Since innovation is profitable for large firms, the increase in demand generated by trade explains the large observed investment rates.

3.1 States and actions

Each firm is defined along four dimensions. The size represented by the capital stock ($K \in \mathcal{K}$), its productivity level ($\omega \in \Omega$), whether it is an R&D firm ($R \in \{0, 1\}$) and whether it is an incumbent or a potential entrant ($\chi \in \{0, 1\}$). We can now define the state of firm i in period t , s_{it}

$$s_{it} = (K_{it}, \omega_{it}, R_{it}, \chi_{it})$$

An aggregate state S_t summarizes the state of the industry: average deflated revenues (\tilde{Y}_t is industry revenues and \tilde{P}_t is the industry price)

$$S_t = \frac{\tilde{Y}_t}{\tilde{P}_t}$$

To fit the model statistically I specify unobserved heterogeneity. Payoff shocks are privately observed by the firm, unobserved by the econometrician and include shocks to investment costs ε_{it}^I , to the sunk cost ε_{it}^{RD} , to the scrap value ε_{it}^{scrap} , and to profits ε_{it}^π . The distribution of these shocks is nonparametrically unidentified (see section 4.4). The shocks have to satisfy conditional independence (Rust (1994)). Thus, I will assume the vector of payoff shocks $\varepsilon_{it} = (\varepsilon_{it}^I, \varepsilon_{it}^{RD}, \varepsilon_{it}^{entry}, \varepsilon_{it}^{scrap}, \varepsilon_{it}^\pi)$ are independently drawn with distribution, $F(\varepsilon)$.

²The models of Hopenhayn (1992) or Melitz (2003) remove any aggregate uncertainty to obtain a deterministic equilibrium.

Firms' available choices are the investment level ($I \in \mathbb{R}_+$) which adds to the current capital stock and R&D ($R \in \{0, 1\}$) which leads to productivity improvements. Firms can also decide to enter or exit ($\chi \in \{0, 1\}$). The choices are summarized in the vector of actions a_{it}

$$a_{it} = (I_{i,t+1}, R_{i,t+1}, \chi_{i,t+1})$$

For convenience I will sometimes use the short notations for the vector of states and actions (s_{it}, a_{it}) and in other occasions I use the individual states and actions.

3.2 State transition

Individual states evolve over time. Most naturally, the evolution is stochastic for productivity and deterministic for capital.

Productivity R&D firms have better prospects for their productivity than non-R&D firms (in a probabilistic sense). Unless firm level prices are observed, we cannot separate "true" productivity from price margins. In the absence of observed firm level prices, I adopt the broad definition for productivity.

The *internal* source of uncertainty distinguishes R&D investment from other decisions like capital investment, labor hiring, entry and exit which have deterministic outcomes and where the sources of uncertainty are *external* to the company (*e.g.* due to the environment, competition, demand, etc). Productivity follows a controlled first order Markov process.

$$\begin{aligned} \omega_{i,t+1} &= E(\omega_{i,t+1} | \omega_{it}, R_{it}) + \nu_{i,t+1}^\omega \\ &= P^{\cdot,\omega}(\omega_{it}, R_{it}) + \nu_{i,t+1}^\omega \end{aligned} \tag{1}$$

where ν_{it}^ω is independently and identically distributed across firms and time and $P^{\cdot,\omega}(\cdot)$ is a parametric function used to approximate the conditional expectation function. The transition for individual productivity is estimated separately for R&D and non-R&D firms and several functional forms for $P^{\cdot,\omega}(\cdot)$ will be reported, including polynomial functions of order n .

Capital stock The capital stock depreciates at rate δ and investment adds to the stock with transition

$$K_{i,t+1} = (1 - \delta)K_{it} + I_{i,t+1}$$

R&D R&D is an absorbing state and the sunk cost is paid only once. A non-R&D firm ($R_{it} = 0$) can decide to do R&D ($R_{is} = 1$) and this will remain for all $s \geq t + 1$. R&D expenditures can be seen as a technological upgrading with a permanent effect. I can relax this assumption and allow an R&D transition where firms can switch in and out of R&D. In the sample, firms rarely switch in and out of R&D (see Tables II and III) and to relax the absorbing state assumption I would need a longer time series in order to identify the switching costs. Furthermore, we are interested in understanding the impact of trade liberalization on R&D behavior and the permanent shift is sufficient to capture these effects.

Entry/exit A potential entrant can enter and an incumbent remain in the market, $\chi_{i,t+1} = 1$. Alternatively, an incumbent can exit and a potential entrant can stay out, $\chi_{i,t+1} = 0$. Exiting or staying out are also absorbing states.

3.3 Payoffs

Firms compete with the other firms to sell their products. The industry is very fragmented and each firm is small (monopolistic competition). The profits obtained are the solution to the static pricing game and competition is summarized by the aggregate state, S_t . The profit function, π , is bounded and additively separable in the payoff shocks

$$\pi(s_{it}, S_t, a_{it}, \epsilon_{it}) = \pi(s_{it}, S_t, a_{it}) + \epsilon_{it}(a_{it}) \quad (2)$$

There are several adjustment costs to consider.

Investment cost Lumpy investment behavior at the plant level is well documented. It can be rationalized by the existence of strong non-convex adjustment costs and irreversibilities. However, the disinvestment reported in the sample (at book value) contains too much noise and there are few (39) observations with zero investment. Thus, there is

insufficient variation in the data to allow identification of a very flexible adjustment cost function and I will adopt quadratic adjustment costs with total irreversibility.

$$C^K(I_{it}, K_{it-1}) = \left[\mu_1 I_{it} + \mu_2 \frac{I_{it}^2}{K_{it-1}} \right] + \varepsilon_{i,t-1}^I I_{it} \quad \text{if } I_{it} \geq 0 \quad (3)$$

where μ_2 indexes the degree of convexity. This simple parametrization still captures the important investment features: option value of investment and periods of inaction. I also report estimates for the model where the quadratic term is set to zero and the results show that the estimates of the remaining parameters are not affected. A more general form for the investment function is possible at the cost of increasing the number of parameters to estimate.

R&D costs R&D decisions take the form of projects implemented over a given period - technological upgrading, catchup innovation or introduction of new technologies. If the whole project is not implemented the returns are likely to be small while they become permanent upon completion. Sunk costs are a good first order approximation to the underlying R&D cost structure. We can write them as $\lambda + \varepsilon_{it}^{RD}$ where λ is the average sunk cost and ε_{it}^{RD} is a private cost shock. Not modeling current expenditures (variable or fixed costs) allows me to avoid considering an extra policy function. As mentioned above, the evidence supports a sunk cost specification.³

Entry cost Potential entrants are short lived and cannot delay entry. Upon entry they must pay a (privately observed) sunk entry fee of $E + \varepsilon_{it}^{entry}$ to get a productivity/capital draw from the distribution $p^E(\omega_{t+1}, K_{t+1} | \chi_t = 0)$. Since entry effects are captured in the equilibrium transition for the aggregate state, it is not necessary to model entry for the estimation. When the equilibrium is calculated in the analysis of counterfactuals, the entry process will be explicitly modeled.

Exit value Every period each firm has the option of exiting the industry and receive a scrap value of $e + \varepsilon_{it}^{scrap}$. Changes to how exit is modeled have only marginal effects on the value function and consequently on the estimates of investment and sunk costs.

³Sunk, fixed and variable costs could be identified. The sunk costs would be identified off the first time it was decided to do R&D, fixed costs would be identified off the switches in and out of R&D and finally the variable costs would be identified off the variations in observed R&D expenditures. In practice, this can only be achieved if there is sufficient variation in the R&D data and a long time series.

Payoff function Let $\theta = (\mu_1, \mu_2, \lambda, e)$ be the vector of cost parameters. Using the specified cost structure the per period return function of an incumbent is

$$\begin{aligned} \pi(\omega_{it}, K_{it}, R_{it}, S_t, R_{i,t+1}, \chi_{i,t+1}, I_{i,t+1}, \boldsymbol{\varepsilon}_{it}; \alpha, \theta) &= \\ &= \tilde{\pi}(\omega_{it}, K_{it}, S_t, \alpha, \varepsilon^\pi) + \varepsilon_{it}^\pi - \mu_1 I_{i,t+1} - \mu_2 \frac{I_{i,t+1}^2}{K_{it}} - \varepsilon_{it}^I I_{i,t+1} \\ &\quad - (\lambda + \varepsilon_{it}^{RD})(R_{i,t+1} - R_{it})R_{i,t+1} + (1 - \chi_{i,t+1})(e + \varepsilon_{it}^{scrap}) \end{aligned} \quad (4)$$

Where $\tilde{\pi}(\cdot)$ is the gross profit function (gross of adjustment costs) and the payoff shocks enter additively (Rust (1994)).

There are two options to specify the gross profit function, $\tilde{\pi}(\cdot)$. First we could parametrize demand and the production function and write the reduced form revenues as the solution to the static pricing game. Examples that are consistent with the aggregate state specification are monopolistic competition or symmetric Cournot competition. Alternatively, when gross profits are observed, the profit function can be left unspecified and estimated directly. The second route is followed here. Reported cash flows are used to estimate the profit function. Since cash flows can take negative values, I allow for fixed production costs

$$\tilde{\pi}(\omega_{it}, K_{it}, R_{it}, S_t, \alpha) = \alpha_0 e^{\alpha_1 \omega_{it}} K_{it}^{\alpha_2} S_t^{\alpha_3} + \alpha_4 + \alpha_5 K_{it} + \alpha_6 R_{it} \quad (5)$$

where the vector of parameters, $\alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$ is estimated by nonlinear least squares.

3.4 Value function

We can finally write down the dynamic problem faced by an individual firm. Players choose actions to maximize the discounted sum of profits

$$\begin{aligned} V(s_{it}, S_t) &= \max_{\{a_{is}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \pi(s_{is}, S_s, a_{is}, \boldsymbol{\varepsilon}_{is}) \\ &= \max_{a_{it}} \pi(s_{it}, S_t, a_{it}, \boldsymbol{\varepsilon}_{it}) + \beta E_t^{a_{it}} V(s_{i,t+1}, S_{t+1}) \end{aligned}$$

where the expected continuation value is integrated over the possible values of the state variables conditional on the current state and actions

$$E_t^{a_{it}} V(s_{i,t+1}, S_{t+1}) = \int_{s_{i,t+1}} \int_{S_{t+1}} V_{i,t+1} p(a_{it}, s_{it}, s_{i,t+1}) q(s_{it}, S_t, S_{t+1}) ds_{i,t+1} dS_{t+1}$$

the transition $p(a_{it}, s_{it}, s_{i,t+1})$ is the probability of reaching state $s_{i,t+1}$ from state s_{it} when action a_{it} is chosen, and the function $q(s_{it}, S_t, S_{t+1})$ is the probability of reaching state S_{t+1} from state (s_{it}, S_t) . The individual transition ($p(\cdot)$) is a primitive of the model assumed known to all players, whereas the industry state transition ($q(\cdot)$) represents the beliefs about the evolution of the industry state.

Equilibrium In equilibrium, beliefs $q(s_{it}, S_t, S_{t+1})$ are consistent with optimal behavior (see the Technical Appendix) . The (stochastic) *equilibrium* transition for the aggregate state is $q^*(s_{it}, S_t, S_{t+1})$.

4 The estimation procedure

The estimation is done in three steps. In the first step, I estimate unobserved productivity. In the second step I estimate the parameters, α , of profit function ($\tilde{\pi}(\omega_{it}, K_{it}, R_{it}, S_t, \alpha)$), the firm-level and industry-level state transitions, ($p(\omega_{i,t+1} | \omega_{it}, R_{it}, \chi_{it})$ and $q(S_t, S_{t+1})$) and the equilibrium policy functions for investment ($I(\omega_{it}, K_{it}, R_{it}, S_t)$), R&D ($R(\omega_{it}, K_{it}, R_{it}, S_t)$) and exit ($\chi(\omega_{it}, K_{it}, R_{it}, S_t)$).⁴ In the third step the dynamic parameters (μ_1, μ_2, λ, e) are recovered using the equilibrium conditions. Since the model has no analytical solution, I use computational methods.

4.1 Step 1: Productivity

Total factor productivity is an unobserved state variable that can be estimated as the residual from a production function.⁵ To estimate the production function I adopt the method proposed by De Loecker (2007) that can be used in situations where there is imperfect competition and revenues are observed instead of quantities (see also Klette and Griliches (1995)). The method was developed for single agent models and I propose a

⁴Assuming players own effect on the aggregate state is negligible we can write $q(s_{it}, S_t, S_{t+1}) = q(S_t, S_{t+1})$. This is the case when players are infinitesimal.

⁵Akerberg et al. (2007) provide an excellent survey of the literature for estimating production functions.

slight variation that can be applied to equilibrium models. The difference in equilibrium models is that the input demand function depends on the industry state, more precisely, the aggregate state (see Appendix A.2 for a discussion).⁶

4.2 Step 2: Policies and transitions

4.2.1 Policies

The policy functions are estimated using the observations for the state variables and actions. The optimal solution to the dynamic problem faced by an individual firm is the investment function which I estimate separately for R&D and non-R&D firms using a flexible polynomial

$$\frac{I_{i,t+1}}{K_{it}} = P^{n,i}(\omega_{it}, K_{it}, S_t, R_{it}) + \varepsilon_{it}^I \quad (6)$$

A lower choice of n to approximate the polynomial function $P^{n,\cdot}(\cdot)$ is preferred. Higher order polynomials can create large distortions and generate a poor approximation (e.g. Runge's phenomenon). This is more likely to occur in intervals of the data with less observations that are normally at the tails of the distribution. Since the estimation procedure averages over the states, the large distortions in the policy functions' estimates at the tails can create a large bias in the averaged results. As a further check on the goodness of the approximations, I evaluate how well the estimated policies match the data by comparing the simulated predictions with the actual behavior.

The probability of doing R&D is

$$\Pr(R_{i,t+1} = 1 | R_{it} = 0, s_{it}, S_t) = \Phi \left(-\lambda + \beta \begin{bmatrix} E\{V(s_{i,t+1}, S_{t+1}) | R_{i,t+1} = 1\} \\ -E\{V(s_{i,t+1}, S_{t+1}) | R_{i,t+1} = 0\} \end{bmatrix} \right)$$

which I approximate by

$$\Pr(R_{i,t+1} = 1 | R_{it} = 0) = \Phi \left(P^{n,rd}(\omega_{it}, K_{it}, S_t, R_{it} = 0) \right) \quad (7)$$

⁶There is also the selection problem resulting from exit of less productive firms from the sample. The correction for selection is negligible.

The same argument in favor of lower order polynomials is in place here. Finally, the exit function is treated similarly

$$\Pr(\chi_{i,t+1} = 0 | \chi_{it} = 1) = \Phi(P^{n,x}(\omega_{it}, K_{it}, S_t, R_{it})) \quad (8)$$

Equation (6) is estimated by OLS and equations (7) and (8) by maximum likelihood.

4.2.2 The transition function

There are two stochastic transition functions to consider: productivity and the aggregate state. The productivity transition is a primitive of the model, specified in equation (1) and can be estimated by OLS. The transition for the aggregate state is an equilibrium object resulting from optimal behavior but can be estimated directly from the data. I parametrize it as

$$\ln(S_{t+1}) = \mu_{SS} + \rho_S \ln(S_t) + \nu_{t+1}^S \quad (9)$$

where ν_{t+1}^S is a zero mean normal random variable with variance $\sigma_{SS}^2 = \sigma_S^2(1 - \rho_S^2)$. The variance of the aggregate state represents the aggregate uncertainty affecting investment. The intercept is $\mu_{SS} = (1 - \rho_S)\mu_S$, and $(\mu_S, \sigma_S, \rho_S)$ are the unconditional mean, variance and autocorrelation for the $\ln(S)$ process.

4.3 Step 3: Minimum distance estimator

Using the estimated profits, policies and transition functions, the cost parameters are finally recovered as follows:

1. Choose n_s different starting values $\{s_{it}^{(j)}, S_t^{(j)}\}_{j=1}^{n_s}$ for $t = 0$. For example the starting values can be the observations in the sample;
2. For each value $(s_{it}^{(j)}, S_t^{(j)})$, draw a vector of payoff shocks, $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{it}^I, \varepsilon_{it}^{RD}, \varepsilon_{it}^{scrap}, \varepsilon_{it}^\pi)$.
3. Calculate actions $(a_{it}^{(j)})$ at each state $(s_{it}^{(j)}, S_t^{(j)})$ using the estimated policy functions and the payoff shocks drawn in 2.;

4. Draw shocks to productivity ($\nu_{i,t+1}^\omega$) and to the aggregate state (ν_{t+1}^S) and update states ($s_{i,t+1}^{(j)}, S_{t+1}^{(j)}$) using the shocks drawn, the estimated transition functions and the actions calculated in 3.;
5. Repeat steps 2. to 4. for \bar{T} periods, and construct a sequence of actions and states: $\{a_{it}(s_{i0}^{(j)}, S_0^{(j)}), s_{it}(s_{i0}^{(j)}, S_0^{(j)}), S_t(S_0^{(j)})\}_{t=1}^{\bar{T}}$. Use this sequence to calculate the discounted stream of profits for a given parameter vector θ and for each $j = 1, \dots, n_s$:

$$V^{(j,m)}(s_{i0}^{(j)}, S_0^{(j)}) = \sum_{t=0}^{\bar{T}} \beta^t \pi(a_{it}, s_{it}, S_t, \varepsilon_{it}; \hat{\alpha}, \hat{P}^n, \theta)$$

6. Repeat steps 2. to 5. for $m = 1, \dots, n_J$ times and calculate an average estimate for the expected value at each (j) state:

$$\widehat{EV}(s_{i0}^{(j)}, S_0^{(j)}; \tilde{\alpha}, \theta) = \frac{1}{n_J} \sum_{m=1}^{n_J} V^{(j,m)}(s_{i0}^{(j)}, S_0^{(j)})$$

The equilibrium conditions imply that at equilibrium beliefs, $q^*(\cdot)$, strategy $a(\cdot)$ is an equilibrium if for all $a' \neq a$ the following condition holds:

$$V(s_{i0}, S_0; a, q^*(S_{t+1}|S_t); \theta) \geq V(s_{i0}, S_0; a', q^*(S_{t+1}|S_t); \theta)$$

Given the linearity of the value function in the dynamic parameters we can write

$$V(s_{i0}, S_0; a, q^*(S_{t+1}|S_t); \theta) = W(s_{i0}, S_0; a, q^*(S_{t+1}|S_t)) * \theta$$

where $\theta = [1, \mu_1, \mu_2, \lambda, e]$, $W(s_{i0}, S_0; a, q^*(S_{t+1}|S_t)) = E_{a|s_{i0}, S_0} \sum_{s=0}^{\infty} \beta^s w_{is}$ and $w_{is} = [\tilde{\pi}(s_{is}, S_s; \alpha), I_{is}, I_{is}^2, \mathbf{1}(R_{is+1} = 1, R_{is} = 0), \mathbf{1}(\chi_{is+1} = 0, \chi_{is} = 1)]$;⁷

7. Calculate the difference between the optimal and non-optimal value function for each policy/state pair $(X_k, k = 1, \dots, n_I)$, where $X_k = (a'_{it}, s_{i0}, S_0)$ and there are

⁷When the paid sunk cost is $\lambda + \varepsilon_{is}^{RD}$, the conditional expected value is $\tilde{\lambda} = \lambda + E[\varepsilon_{is}^{RD} | R_{is+1} = 1, R_{is} = 0]$. The unconditional is larger than the conditional expected value and using linearity we can only recover the later since the standard deviation of ε^{RD} is not known (and cannot be identified). What is identified is the average cost paid by a firm that decided to do R&D. We can easily incorporate the conditional expected sunk cost in the policy simulations and the conclusions should be interpreted accordingly. The same holds for the exit value.

$n_I = n_a * n_s$ such pairs:

$$\hat{g}(X_k; \theta, \hat{\alpha}, \hat{P}^n) = \left[\hat{W}(s_{i0}, S_0; a, q(S_t, S_{t+1})) - \hat{W}(s_{i0}, S_0; a', q(S_t, S_{t+1})) \right] * \theta$$

The equilibrium condition implies that estimated policies are optimal and the expected value of using strategy a should not be smaller than using alternative a' . A violation of equilibrium conditions occurs when $\hat{g}(X_k, \theta, \hat{\alpha}, \hat{P}^n) < 0$. The empirical minimum difference estimator minimizes⁸ the squared violations of such equilibrium conditions

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \hat{J}(\theta; \hat{\alpha}) = \arg \min_{\theta \in \Theta} \frac{1}{n_I} \sum_{k=1}^{n_I} \left(\min \left\{ \hat{g}(X_k; \theta, \hat{\alpha}, \hat{P}^n), 0 \right\} \right)^2$$

I set the time of each path at $\bar{T} = 75$, the discount factor at $\beta = 0.96$, the number of starting configurations is the total number of observations ($n_s = 1,017$), the number of simulations for each configuration $n_J = 200$ and the number of alternative policies $n_a = 125$, giving a total number of "differences", $n_I = 127,125$.

Choice of alternative policies The vector of cost parameters, θ , must rationalize the estimated strategy profile, \hat{a} . In general, θ can be point or set identified, depending on the model, available sample and alternative policies. Bajari et al. (2007) also propose an estimation method when the parameters are set identified.

The inequality in the objective function, $J(\theta)$, arises from comparing \hat{a} with alternative policies, a' . The way alternative policies are chosen will influence estimation and identification. If we choose alternative policies very *far* from \hat{a} , the identified set increases and we only achieve parametric identification while if we choose alternative policies very *close* to \hat{a} , the identified set shrinks. Since we can produce as many alternative policies as required, we can improve the estimation by choosing the alternative policies and artificially generating non-optimal observations.

This raises the question of how should we choose the alternative policies? One option is to add a slight perturbation to the estimated policy function. If the perturbation is

⁸When the objective functions is not smooth (*e.g.* problems with discontinuous, non-differentiable, or stochastic objective functions) using derivative based methods might produce inaccurate solutions. Using derivative free methods (for example, the Nelder-Mead algorithm) to minimize the empirical minimum distance (\hat{J}) helps to circumvent these problems. Non-smooth functions occur with finite n_I , because of the min operator in the empirical objective function, \hat{J} . This can create discontinuities even when $g()$ is continuous in θ .

positive, all alternative actions will be larger and the parameters are only identified in a one sided set. For example, if alternative R&D start-up decisions are more frequent (i.e. positive perturbations added to the optimal policy function), the alternative policies will generate high levels of R&D behavior. Firms will do more R&D than actually observed in the sample which can only be rationalized if the sunk costs are high (bounded below). However, the sunk costs are not bounded above unless we also add negative perturbations. I add zero-mean normally distributed errors to the investment, R&D and exit policies with standard deviations of 1, 0.5 and 0.5. This translates in a 95% probability for the alternative policies for investment to be in the interval $\pm 200\%$ from the estimated one. I also evaluate the sensitivity of the estimates to other choices of a' .

4.4 Identification

Identifying restrictions Assuming agent's optimal dynamic behavior, imposes no testable predictions. Without further restrictions, a given reduced form (observed) model can be rationalized by more than one parametric form for the structural model. The identification problem in dynamic models is well known (Rust (1994), Magnac and Thesmar (2002), Pesendorfer and Schmidt Dengler (2008) and Bajari et al. (2008)).

The unknown structural objects are the period returns, distribution for the shocks, state transition function and discount factor: $\pi(a_{it}, \mathbf{s}_t, \boldsymbol{\varepsilon}_{it}), F(\boldsymbol{\varepsilon}_{it}), p(s_{i,t+1}|s_{it}, a_{it}), \beta$. These objects cannot be separately identified. Pesendorfer and Schmidt Dengler (2008) extend the work of Magnac and Thesmar (2002) on single agent models to dynamic games and provide one solution to the identification problem. One solution is to normalize the period returns for some outside alternative and to use exclusion restrictions (i.e. state variables that can be excluded from the period returns but enter the policy functions). Even so, without further restrictions the discount factor and distribution of cost shocks are nonparametrically unidentified. There are also other alternatives. One is the use of parametric restrictions on the return function. Another alternative is to estimate part of the return function directly when some measure of returns is observed.

Out of the four structural objects to be estimated, I use observed cash flows to estimate part of the return function ($\tilde{\pi}(\cdot; \alpha)$) in the second step. Assuming agents have rational expectations, I recover their beliefs by estimating the evolution for the states in the second step. The discount factor and the distribution of shocks ($\beta, F(\varepsilon)$) are set exogenously.

The only object left to estimate is the cost function. This function takes the value of zero when there is inaction (no investment and no R&D start-up), $C(s_{it}, a_{it} = 0) = 0$ and is unaffected by productivity and the aggregate state. Thus, the cost function satisfies both the normalization and exclusion restrictions.

Sample features The cost function is identified from simple sample features. The cost parameters are estimated to rationalize the observed choices given the observed returns and state transitions. For example, the estimated sunk costs compare the profits earned by the firms that decided to do R&D at a given state with the profits of the firms that decided not to do R&D. Had these costs been higher, we would have observed less R&D and had these costs been lower, we would have observed more R&D.

Beliefs Firms operate in a dynamic and uncertain environment and have to form beliefs about the future. In general, these beliefs are not known or even estimable by the econometrician. For this reason, I maintain the hypothesis of full rationality and match the beliefs to the observed behavior. This assumption can only be relaxed in some particular cases. For example, if firms use a forecasting method that is known to the econometrician, we can try to estimate the beliefs from the data. Full rationality is just one particular forecasting method that we can estimate from the data. Thus, any solution has to make an assumption about belief formation.⁹

5 Results

Estimation is done in three steps. An evaluation of the sensitivity to errors and bias in the profits, policies and transitions is also conducted. As a robustness check I also try alternative polynomials in the second step, different static profit functions and different productivity measures. Finally, alternative specifications for the dynamic cost parameters are also reported.

⁹An alternative recently explored in Aradillas-Lopez and Tamer (2009) is the use of rationalizability to derive bounds on the parameters by using weaker concepts than full rationality. Fershtman and Pakes (2009) also provide some extensions to the concept of rationality.

5.1 First step

5.1.1 Productivity

Total factor productivity is calculated as the residual from a production function. The method is discussed in Appendix A.2. Three specifications for the transition function $E(\omega_{it}|\omega_{it-1}, R_{it-1})$ are reported: linear and cubic polynomials and a sigmoidal function. All three yield similar results and share the same relevant feature - larger productivity for R&D firms.

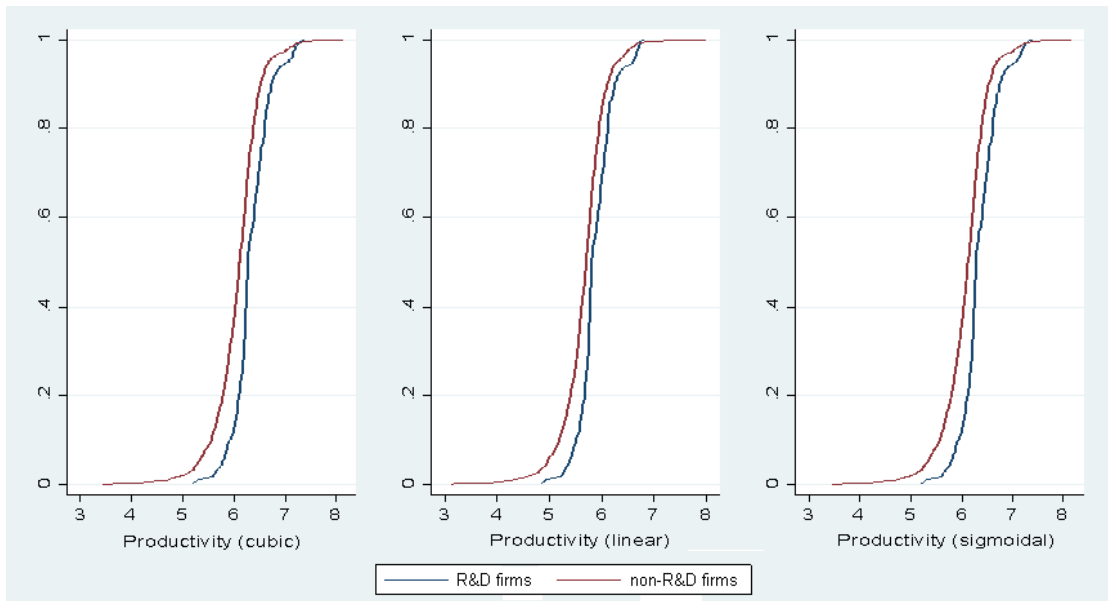


Figure 1: Productivity distribution (cubic, linear and sigmoidal approximation)

Firms are willing to pay the sunk cost of innovation if they expect a gain in the future, in this case, larger productivity. Estimated productivity for R&D firms is on average 26% larger than for no-R&D firms. (Figure 1)

5.2 Second step

5.2.1 Static profits

The gross profit function in equation (5) is estimated using reported cash flows. The capital elasticity of profits reported in Table IV is 0.74 (0.65 if we include the capital fixed cost component) consistent with diminishing returns on capital. The productivity elasticity of profits is 0.95 (1.23 if we include the capital fixed cost component) while

	(i)		(ii)		(iii)		(iv)		(v)	
Dependent Variable:	Cash flows $\tilde{\pi}_{it}$									
	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>
$\hat{\alpha}_0$	-5.03	1.21	-4.02	1.08	-6.44	0.85	-6.48	0.08	-6.79	0.88
$\hat{\alpha}_1$	0.95	0.16	0.80	0.15	1.22	0.04	1.23	0.04	1.25	0.04
$\hat{\alpha}_2$	0.74	0.04	0.76	0.04	0.64	0.02	0.65	0.02	0.66	0.02
$\hat{\alpha}_3$	0.14	0.05	0.13	0.05	0.19	0.06	0.19	0.06	0.18	0.06
$\hat{\alpha}_4^a$	-8.65	13.47	-7.19	13.82	-2.45	0.14			-5.94	13.29
$\hat{\alpha}_5$	-0.12	0.08	-1.98	1.03						
$\hat{\alpha}_6^a$	109.5	23.3							115.8	23.1
R^2	87%		87%		87%		87%		87%	

Notes: ^aCoefficients and standard errors divided by 1,000. Results for the profit function (Equation (5)) using different specifications in columns (i) to (v).

Table IV: Reduced form profit function estimates (Equation (5)).

the elasticity of profits to the average size of competition is 0.14. Finally, the fixed cost component is increasing for firms with larger capital stocks and a 100,000 Euros increase in capital stock leads to a 15,000 Euro increase in fixed costs. On average, R&D firms earn 105 thousand Euros more over the productivity gains.

Overall the results illustrate the importance of both physical capital and productivity as determinants of profitability. The fit is quite good and over 85% of the total variation in profits is explained by the state variables. Since both variables are quite persistent (see Table V), they characterize the persistence in profits.

5.2.2 Transition function

Aggregate state The aggregate state transition specified in equation (9) is fully specified by three parameters: the mean, variance and autocorrelation of the aggregate state. The estimated sample values are $\hat{\mu}_S = 13.18$, $\hat{\sigma}_S = 0.28$ and $\hat{\rho}_S = 0.79$.

Productivity Productivity transition in equation (1) is estimated separately for the R&D and non R&D firms. Two parametric specifications are reported in Table V and the results with four specifications are plotted in Figure A.3. Two main features emerge: strong persistence in productivity across all specifications and mean reversion. These two features are absent in models with static productivity and suggest that productivity converges slowly. Models with static productivity will thus overestimate the value of firms with high current productivity and underestimate the value of firms with low current productivity.

Dependent Variable: Productivity ($\hat{\omega}_{it}$)	(i)		(ii)		(iii)		(iv)	
	Non-R&D		R&D		Non-R&D		R&D	
	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>
$\hat{\omega}_{i,t-1}$	-3.55	1.06	-11.54	5.91	0.92	0.01	0.95	0.02
$\hat{\omega}_{i,t-1}^2$	0.72	0.18	1.92	0.92				
$\hat{\omega}_{i,t-1}^3$	-0.04	0.01	-0.10	0.05				
const.	9.66	2.05	27.31	12.58	0.48	0.09	0.32	0.13
\hat{R}^2	85%		93%		84%		93%	
Obs.	790		254		790		254	
Firms	197		59		197		59	
$s.\hat{e}.(\nu^\omega)$	0.19		0.11		0.20		0.12	

Notes: Results for the productivity transition using a 3rd order polynomial are reported in columns (i) and (ii) and for the linear case in columns (iii) and (iv).

Table V: Productivity transition estimates.

After evaluating the different approximations we conclude that the 6th order polynomial performs poorly at the tails of the distribution. For the R&D firms, the results for the linear and cubic polynomials are similar, while the sigmoid function reveals problems of non-stationarity in the region with low productivity. For the non-R&D firms, the 6th order polynomial and the sigmoid function also reveal problems of non-stationarity. The linear case does not provide a good fit in the low productivity region while the cubic polynomial fits the data well both in terms of average fit ($\hat{R}^2 = 85\%$) and at the tails of the distribution.

5.2.3 Investment, R&D and Exit policies

Estimates for the policy functions are reported in Table VI while Figures A.3 to A.4 in the Appendix report the data and fit projected in a two-dimensional plane for productivity and capital. The R&D function in equation (7) is estimated with a probit model, and the investment function in equation (6) is estimated by ordinary least squares. Due to the limited number of observations, a linear probit is used for the exit policies. Again, the preference for low order polynomials is related with the poor fit of the high order polynomials in regions with few observations (particularly at the tails) and the resulting problem that averages are very sensitive to incorrect predicted actions at the extremes. Also, in some situations we would like the fitted functions to preserve basic properties,

like monotonicity.

Dep. Var.: Model:	(i)		(ii)		(iii)				(iv)		(v)	
	R&D Probit		R&D Probit		Investment OLS				R&D firms		Exit Probit	
	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Non R&D firms Coef.	<i>s.e.</i>	R&D firms Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>
s_{t-1}	-0.07	<i>0.22</i>			-0.20	<i>0.16</i>	-0.09	<i>0.33</i>	0.07	<i>0.46</i>		
$k_{i,t-1}$	2.09	<i>1.07</i>	2.08	<i>0.91</i>	-0.24	<i>0.55</i>	-1.28	<i>1.82</i>	0.00	<i>0.11</i>		
$k_{i,t-1}^2$	-0.12	<i>0.05</i>	-0.07	<i>0.03</i>	0.00	<i>0.02</i>	0.24	<i>0.08</i>				
$\hat{\omega}_{i,t-1}$	-2.23	<i>2.53</i>	0.08	<i>0.22</i>	1.32	<i>1.65</i>	14.50	<i>6.28</i>	-0.51	<i>0.30</i>		
$\hat{\omega}_{i,t-1}^2$	-0.01	<i>0.25</i>			-0.20	<i>0.13</i>	-0.35	<i>0.68</i>				
$\hat{\omega}_{i,t-1} * k_{i,t-1}$	0.18	<i>0.19</i>			0.14	<i>0.06</i>	-0.71	<i>0.36</i>				
Const.	-8.77	<i>9.05</i>	-16.76	<i>6.21</i>	4.25	<i>6.19</i>	-31.69	<i>17.54</i>	-0.39	<i>6.18</i>		
\hat{R}^2	24.2%		23.9%		40%		55%		7.4%			
Obs.	838		838		801		204		1044			
Firms	212		212		208		51		223			

Notes: Results for the R&D start-up probit regression are reported in columns (i) and (ii) and for the investment OLS regressions in columns (iii) and (iv). Finally, results for the exit probit regression are reported in column (v).

Table VI: Policy function estimates: R&D, investment and exit.

Exit As expected, larger and more productive firms are less likely to exit. However, the small number of observations for exit make the results statistically insignificant. Alternatively we could fit an exogenous exit probability with aggregate moments. Doing so, would not account for selection due to exit and overestimate the value of firms with low productivity, and vice-versa. The effect on the value function of the two approaches is marginal.

R&D probit There are strong and significant size effects. Small firms have a probability of doing R&D close to zero, which increases strongly with capital and levels out for large capital stocks (Figure A.2). The strong size effect supports evidence for the potential gains from trade. However, conditional on size there is no evidence that more productive firms are more likely to do R&D. The non significant productivity effect suggests the selection of more productive firms into R&D is not meaningful.

Investment For non-R&D firms, investment is increasing in both size and productivity with a strong coefficient on the interaction. For R&D firms, the size effect is milder

while the productivity effect is stronger. This suggests size-driven investment for the most productive non-R&D firms, up until the optimal level. Once the optimal level is reached and R&D is started, the size effect becomes less pronounced and productivity is the main driver of investment. One puzzling feature is the insignificance of the aggregate state.

Summarizing, investment of non-R&D firms is strongly influenced by size and productivity. The R&D decision is affected by size but not by productivity. Finally, large and more productive firms are less likely to exit, although none of the coefficients in the exit probit is significant.

Implications The results suggest the following model. Relatively more productive firms start by investing in physical capital. Once they reach a certain size, R&D becomes a profitable investment. After R&D is done, productivity increases and since firms are now larger, investment *rates* decrease. The same mechanism is suggested by the descriptive statistics (Table A.I). Growth rates are larger for non-R&D firms (for revenues, value added and labor productivity) and investment rates are lower for R&D firms. Furthermore, R&D firms are larger and more productive. Even though labor productivity *growth* is larger for non-R&D firms, labor productivity *levels* are larger for the R&D firms. Taken together, these features characterize firm level dynamics and motivate the use of a dynamic model with productivity and capital investment.

5.3 Third step

The minimum distance estimator outlined above is now implemented to recover the dynamic parameters reported in the first panel (Model 1) of Table VII: the linear and quadratic investment cost, R&D sunk cost and exit value. Confidence intervals were constructed using the bootstrap.¹⁰

The values are estimated with the expected signs. Sunk costs are estimated at about 3.4 million Euros, almost two times the average revenues and more than one year worth of revenues for an average R&D firm. Investment costs are increasing and convex but they

¹⁰Step 3 only produces simulation error. This error is reduced as the number of simulation draws increase ($n_J \rightarrow \infty$). Since bootstrapping requires intense computations, the number of simulations is kept within a manageable value and set $n_J = 125$. The bootstrapped confidence intervals will thus contain some noise from the simulations.

Parameters	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	\hat{e}	\hat{F}
Model 1 - baseline					
Coefs	1.08	3.02	3,369,600	1,606,800	-
5th percentile	-5.29	0.05	-1,423,855	-5,163,630	-
95th percentile	1.66	48.73	16,657,150	9,788,270	-
Model 2 - linear investment costs					
Coefs	1.61	-	3,896,300	993,400	-
5th percentile	0.95	-	-3,105,840	-4,857,300	-
95th percentile	3.35	-	23,086,895	9,222,970	-
Model 3 - operating fixed costs					
Coefs	1.61	-	3,740,000	110,450,200	-4,874,290
5th percentile	0.95	-	-3,542,330	-692,556,855	-30,613,683
95th percentile	3.38	-	28,332,270	490,108,195	20,868,684

Notes: Estimates for the cost parameters and bootstrapped confidence intervals.

Table VII: Cost parameters estimates.

are only statistically significant when we set the quadratic component to zero. The exit value is estimated at around 1.8 million Euros but due to the small number of exits this estimate is very imprecise. Alternative specifications where the quadratic investment cost term, μ_2 , is dropped or a fixed operating cost, F , added are also reported in the second and third panels (Models 2 and 3). Overall the precision of the linear cost improves when we drop the quadratic adjustment cost. The estimate of 1.6 Euros for each Euro of investment suggests indirect investment costs are about 60%.

When I introduce a fixed cost in Model 3, the exit value becomes unrealistically large. This result nicely illustrates the importance of the identification conditions. In this case, the normalization condition is violated making it difficult to separately identify \hat{F} and \hat{e} .¹¹ Estimating the fixed operating cost in the first step using observed profits is a better approach since the profits for the outside alternative (i.e. not exiting) are then normalized.

How reasonable are the estimated parameters? We can compare average profits earned by an R&D firm in the period before it started doing R&D with the average profits earned afterwards. The difference is 230,000 Euros. The value of this difference discounted over a 40 year horizon is 4.6 million Euros, slightly above the estimated sunk costs from

¹¹The estimated values of $\hat{e} = 110,450,200$ of model 3 and $\tilde{e} = 1,606,800$ of model 1 with $\beta = 0.96$ rationalize a "net" value (i.e. $(1-\beta)(\hat{e}-\tilde{e}) = 4,353,736$) close to the fixed cost estimate, $\hat{F} = 4,874,290$. If identification is not possible, there will be an infinite combination of pairs (\hat{F}, \hat{e}) that can rationalize the observed decisions. For example, in a non-stochastic setting, a firm can decide to exit today and collect the exit value \hat{e} or stay one more period and collect $\pi + \hat{F} + \beta\hat{e}$. For an indifferent firm $\pi + \hat{F} + \beta\hat{e} = \hat{e}$. In the absence of fixed costs (F), the indifference condition is $\pi + \beta\tilde{e} = \tilde{e}$. Replacing and solving the two conditions we get $\hat{e} - \tilde{e} = \frac{1}{1-\beta}\hat{F}$.

the structural model. The slight upward bias might be due to a selection effect that is accounted and corrected in the structural model. The estimated sunk costs cannot be compared with reported R&D expenditures. If they could, we would directly use reported values and have no need to estimate them. For example, the sunk costs can be related to internal resources (capital and labor) that have to be allocated to R&D instead of production and will thus not show up in company accounts.

5.4 Robustness

As explained above, bias in the policy function estimates can create a bias in the third step. To evaluate how robust the results are, I estimate the model under alternative specifications for the discount factor, profit function and policies.

Discount factor (Table VIII) Lower discount factors reduce the continuation value and, as expected, decrease the estimated investment and sunk costs. For example, with a discount factor of 0.9 the sunk costs are estimated at 1.2 million Euros and the investment costs at 0.83 for the linear and 1.41 for the quadratic component.

β	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	\hat{e}
<i>0.90</i>	0.83	1.41	1,299,200	-574,500
<i>0.91</i>	0.87	1.58	1,477,800	-504,100
<i>0.92</i>	0.91	1.77	1,702,700	-368,300
<i>0.93</i>	0.95	2.00	1,989,500	-131,900
<i>0.94</i>	1.00	2.27	2,355,300	264,100
<i>0.95</i>	1.04	2.61	2,808,200	835,700
<i>0.96</i>	1.08	3.02	3,368,800	1,607,700
<i>0.97</i>	1.12	3.51	4,028,200	2,731,000
<i>0.98</i>	1.15	4.16	4,355,900	4,733,300
<i>0.99</i>	1.13	5.10	3,042,900	8,481,800

Table VIII: Cost parameters estimates for different discount factors.

Profit function (Table IV) Estimated dynamic cost parameters are also quite robust when the alternative reduced form profit functions from Table IV are used. Overall, the sunk costs are estimated between 1.1 and 3.7 million Euros while the investment costs are only marginally affected.

	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	\hat{e}
Baseline	1.08	3.02	3,369,600	1,606,800
$\alpha_6 = 0$	1.03	3.27	1,111,600	1,156,500
$\alpha_5 = \alpha_6 = 0$	1.16	2.77	1,423,500	1,922,300
$\alpha_4 = \alpha_5 = \alpha_6 = 0$	1.16	2.77	1,403,600	1,946,700
$\alpha_5 = 0$	1.17	2.70	3,672,700	1,979,000

Table IX: Cost parameters estimates for different specifications of the profit function.

Policy functions (Table X) The policy functions play a central role in the estimation. Due to the nonlinearities, even small bias in the estimated policies can get magnified into a large bias in the structural parameters. To evaluate how sensitive the results are, I try simpler (linear) policies. Using a linear investment function, sunk cost are estimated at about 3 million Euros and investment costs at -1.98 for the linear and 29.09 for the quadratic component. Alternatively, using a linear probit for R&D, sunk cost are estimated at 4.5 million Euros and adjustment costs at 1.18 for the linear and 2.01 for the quadratic component. If we allow both the investment and R&D policies to be linear, the sunk cost is estimated at 2.9 millions.

	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$	\hat{e}
Linear R&D probit	1.18	2.01	4,475,100	-992,000
Linear investment	-1.98	29.09	2,987,600	1,174,100
Linear R&D probit and investment	-1.64	26.98	2,920,100	1,489,900

Table X: Cost parameters estimates for different policy functions.

Overall, the results are relatively robust to alternative specifications. A final validation exercise is to evaluate how well the estimated model matches the actual data.

Validation How well does the model fit the evidence? As opposed to other estimation methods, the minimum distance estimator does not match the aggregate moments.¹² Thus, I can use the comparison between the actual data and the simulated aggregate moments as an exercise of cross validation to the model. Before progress is made I must set some parameters: the distribution for the entry cost, the productivity distribution of entrants, market size, wage rate and the standard deviation for the sunk costs. I also set exogenously the average exit value because its estimate is imprecise and too large to rationalize the evidence, generating too much exit.

¹²BBL (2007) also propose a method of moments estimator to reduce the bias in the estimates by averaging out first stage estimation error.

The parameters are set as follows. Mean and variance for the entry costs are set together with the mean and variance for the exit value to fit the average number of firms and entry/exit rates. The values are 1.3 million Euros for the average entry cost and 750 thousand Euros for the exit value with standard deviations of 300 and 100 thousand. The mean and variance for the productivity of entrants is matched with the actual value from the sample: 5.8 and 0.35. Market size is 450 million Euros, similar to the market size in 2004 and the wage rate is 16,000 Euros, also in line with reported average industry wages. The standard deviation for sunk costs is chosen at 337,500 Euros, to match the 30% of R&D firms.

Moments	All	Data		Simulated
		Pre-'97	Post-'97	
<i>Average number of firms</i>	680	536	753	719
<i>Average log-sales</i>	13.18	12.94	13.30	13.11
<i>Percentage of R&D firms</i>	0.21	0.13	0.29	0.30
<i>Average productivity</i>	6.13	6.07	6.18	6.22
<i>St. dev. of productivity</i>	0.45	0.39	0.50	0.47
<i>Average log-capital stock</i>	12.77	12.43	13.00	12.93
<i>St. dev. of log-capital</i>	1.57	1.49	1.58	0.64
<i>Capital-prod. correlation</i>	0.46	0.42	0.47	0.42
<i>Average investment rate*</i>	0.22	0.25	0.20	0.13
<i>St. dev. of invest. rate*</i>	0.23	0.24	0.22	0.22
<i>Entry rate (%)</i>	3.70	5.12	2.30	1.7
<i>Exit rate (%)</i>	-	-	-	1.3

Notes: Sample moments pre and post 1997 and moments for the simulated sample using the estimated parameters.

*Conditional on investment rate smaller than 1.

Table XI: Sample and simulated moments.

Using the full set of parameters (estimated and exogenously set), I solve the model and simulate the industry for 120 periods. To do the simulations I am now required to solve the model and calculate the new equilibrium industry evolution, $q^*(S_t, S_{t+1})$. The computational burden of solving a full dynamic game would be prohibitive while I can solve the proposed aggregate state model in a reasonable time.¹³ After dropping the first 60 periods I calculate the moments for the stationary market structure, reported in Table XI: the mean and standard deviation for productivity, capital, and investment rate, and the correlation between productivity and capital. Also reported are the number of firms,

¹³A MATLAB algorithm to solve the model is available from the author. Solving the model takes about 120 minutes on a 2.6 Ghz Phenom Quad-core computer with 3GB RAM.

aggregate state and entry rate but these values are not free since, as explained, they where "calibrated" by setting the distributions of entry and exit values.

The model can explain well the first and second moments for capital and productivity but generates too little variation in capital. The little variation in capital is probably due to the ex-ante symmetry between firms and no source of persistent unobserved heterogeneity in the model. The model does not provide such a good fit for the investment rates. This is because we observe a high investment transition period. If we consider only the later years (after 2000), investment rates and variation decrease to 0.13 and 0.18, more in line with the predicted values. Finally, the model explains well the correlation between capital and productivity. Using these values as a benchmark, we can now evaluate the effects of some policy changes.

6 Counterfactual Experiments

This section reports the results for a change in market size resulting from trade liberalization and assess the impact on industry R&D, productivity and investment. I also report two other experiments to compare the results: changes in (i) sunk costs of R&D, and (ii) entry costs. Because the model is stylized, the particular numbers generated by the counterfactual simulations should be seen as merely suggestive as the effects of any policy change might depend on other factors not captured by the model.¹⁴

Reduction in market size The goal is to access the models' predictions in case we return to the pre EU integration era, by evaluating the effects of a reduction in market size. Before proceeding I explain why this analysis can be conducted. First, the inexistent internal market allows me to abstract from import competition. Second, I abstract from the strategic effects of trade liberalization on the foreign competition. This is a valid assumption in a monopolistic competition setting where the total market can be divided in individual (national) submarkets. Thus, trade liberalization for this industry can be mimicked by a net increase in market size for home producers. The results are no longer

¹⁴Also, the standard deviation of the payoff shocks is not identified and this object influences the magnitude of the effects. This is particularly relevant for the sunk costs of R&D where we identify the average cost paid by a firm who decided to do R&D and not the average unconditional cost. A decrease in the conditional mean is stronger than an equivalent decrease in the unconditional mean.

valid if there is import competition or any type of strategic interaction between home and foreign producers.

Notice also that since 80% of the firms export I have abstracted from the decision of selling at home or abroad. This shuts down the selection mechanism discussed in the literature resulting from the interaction of heterogeneity with export costs. On the other hand I focus on the effects of heterogeneity interacted with innovation costs. If trade costs were significant so that only a fraction of the firms exported, the selection mechanism would reinforce the mechanism analyzed here.

The results in Table XII suggest that a reduction in market size from 450 million Euros to the level in 1994 of 150 million Euros will cause a reduction in average R&D, capital and productivity. The figures predicted by the model are actually similar to the values observed in the sample for the early periods. Also, entry and exit rates increase and the correlation between capital and productivity decreases. More importantly, however, the number of firms does not decrease linearly with market size. Thus, firms get smaller as market size shrinks. This is the important size effect documented in other studies (*e.g.* Eaton, Kortum and Kramarz (2008)) that is not captured by the models of Melitz (2003) or Eaton and Kortum (2003).

Market size (Y) : $10^6 EUR$	540	495	<i>450</i> <i>base</i>	405	345	285	225	150	105
<i>Average log-sales</i>	13.15	13.13	<i>13.11</i>	13.08	13.06	13.02	12.97	12.92	12.85
<i>Average number of firms</i>	824	771	<i>719</i>	647	557	467	374	250	160
<i>Percentage of R&D firms</i>	0.32	0.31	<i>0.30</i>	0.26	0.25	0.22	0.19	0.14	0.09
<i>Average productivity</i>	6.23	6.23	<i>6.22</i>	6.22	6.21	6.21	6.19	6.17	6.14
<i>St. dev. of productivity</i>	0.47	0.47	<i>0.47</i>	0.47	0.47	0.47	0.47	0.47	0.47
<i>Average log-capital stock</i>	12.97	12.95	<i>12.93</i>	12.88	12.85	12.79	12.73	12.62	12.38
<i>St. dev. of log-capital</i>	0.63	0.64	<i>0.64</i>	0.63	0.65	0.65	0.66	0.73	0.79
<i>Capital-prod. correlation</i>	0.42	0.42	<i>0.42</i>	0.42	0.42	0.41	0.40	0.39	0.39
<i>Average investment rate</i>	0.13	0.13	<i>0.13</i>	0.13	0.12	0.12	0.11	0.09	0.06
<i>St. dev. of invest. rate</i>	0.23	0.23	<i>0.22</i>	0.21	0.22	0.22	0.21	0.23	0.26
<i>Entry rate (%)</i>	1.0	1.3	<i>1.7</i>	2.2	3.1	4.3	6.0	10.3	17.4
<i>Exit rate (%)</i>	0.7	1.0	<i>1.3</i>	1.8	2.7	3.8	5.5	9.8	17.1

Table XII: Predicted moments from changes in market size.

The mechanism causing the size effect illustrates the need of (i) adjustment costs for capital and (ii) aggregate shocks. The first justifies the need of a dynamic model and the second, the need of an equilibrium model with aggregate uncertainty. Models without aggregate shocks (and constant markups) predict market size neutrality.¹⁵ As the

¹⁵An exception here is the work of Melitz and Ottaviano (2008) that allows for varying markups.

simulated results show, this no longer holds in the general case. Aggregate shocks together with adjustment costs for capital create an option value for investment and generate two effects from changes in market size. First, increases in market size push investment up. The equilibrium average capital stock becomes larger which, in the presence of sunk costs, leads to an increase in R&D. Second, as market size increases more firms can fit in the market. In equilibrium, the increase in market size leads to an increase in the number of firms in the market, but because firms are on average larger, the increase is less than proportional. This generates the concave relation between market size and number of firms which is present in Table XII and has been well documented in the industrial organization literature. In the limit we can have Sutton (1998) style effects where industry concentration is bounded below as market size increases.

Reduction in the sunk costs of R&D The sunk costs of R&D play an important role shaping productivity and trade. A reduction of these sunk costs can also promote innovation and trade. Examples of policies to reduce the sunk costs are direct subsidies to R&D start-up or the creation of public R&D labs that could explore economies of scale and avoid duplication costs. The results in Table XIII show that a reduction in sunk costs is expected to lead to an increase in R&D, productivity, capital and investment. More importantly average revenues are increasing which suggests that reducing the sunk costs of R&D can promote firm exports. These effects are similar to an increase in market size and can also rationalize the observed industry change.

Sunk costs (λ) : $10^6 EUR$	2.89	3.06	3.23	3.32	<i>3.40</i> <i>base</i>	3.57	3.74	4.08	4.42
<i>Average log-sales</i>	13.23	13.19	13.13	13.14	<i>13.10</i>	13.06	13.07	13.06	13.05
<i>Average number of firms</i>	1000	991	893	792	<i>714</i>	621	576	563	552
<i>Percentage of R&D firms</i>	0.98	0.88	0.60	0.46	<i>0.29</i>	0.11	0.03	0.01	0.00
<i>Average productivity</i>	6.36	6.34	6.28	6.25	<i>6.22</i>	6.19	6.17	6.17	6.16
<i>St. dev. of productivity</i>	0.42	0.43	0.45	0.46	<i>0.46</i>	0.47	0.47	0.47	0.47
<i>Average log-capital stock</i>	13.44	13.36	13.14	13.06	<i>12.92</i>	12.80	12.76	12.74	12.73
<i>St. dev. of log-capital</i>	0.66	0.65	0.68	0.68	<i>0.63</i>	0.56	0.54	0.52	0.51
<i>Capital-prod. correlation</i>	0.39	0.40	0.44	0.44	<i>0.41</i>	0.37	0.35	0.34	0.33
<i>Average investment rate</i>	0.14	0.14	0.14	0.13	<i>0.13</i>	0.12	0.12	0.12	0.11
<i>St. dev. of invest. rate</i>	0.31	0.29	0.26	0.24	<i>0.22</i>	0.19	0.19	0.18	0.17
<i>Entry rate (%)</i>	0.0	0.1	0.6	1.2	<i>1.7</i>	2.5	2.9	3.0	3.2
<i>Exit rate (%)</i>	0.0	0.0	0.4	0.9	<i>1.3</i>	2.0	2.5	2.6	2.7

Table XIII: Predicted moments from changes in sunk costs of R&D.

Reducing the sunk costs affects only the value of non-R&D firms and leaves the value

of R&D firms unchanged (excluding equilibrium effects). Changes in sunk costs affect the value function through three different mechanisms. First, smaller sunk costs cause a direct reduction in costs. Second, the probability of doing R&D increases with a decrease in sunk costs having an indirect effect and increasing expected benefits. Finally, the equilibrium effects have an opposite effect on the value function. As sunk costs decrease, the value of entering is increased and more firms enter the market leading to an increase in the number of firms. The decrease in sunk costs leads to an increase in R&D and productivity. As a consequence, firms invest more and become larger. The equilibrium effect is not sufficient to counteract the first two effects and the final result of a decrease in the sunk costs is more firms that are both larger and more productive.

Increase in entry costs Lastly, there is often political support for the creation of large firms under the argument that this might spur innovation. Large firms also account disproportionately for total exports. One way to create large firms is by increasing entry costs. Large entry costs will decrease entry rates and protect incumbent firms from competition, allowing them to grow. Larger firms will then be willing to pay the sunk cost and start innovating.

As reported in Table XIV, an increase in entry costs boosts innovation by generating an increase in average size, investment and R&D. However, it also leads to a reduction in the number of firms and entry/exit rates and bad firms are now less likely to exit and be replaced by better firms. Overall, this selection effect might lead to a decrease in average industry productivity, specially if the boost in innovation is not sufficiently strong (or all firms are already innovating). As expected, the increase in entry costs protects firms and leads to an increase in average size.

7 Conclusion

The evidence suggests that technology is endogenous and size is an important driver of technological choices. I have proposed and estimated an equilibrium model with endogenous choices of technology and size for an exporting industry. One disadvantage compared with simpler models is the lack of analytical solutions. I estimate the model for a sample of firms from the Portuguese moulds industry and find evidence of relatively large sunk costs of R&D. These costs are particularly relevant in this industry since firms

Entry costs (e) : $10^6 EUR$	0.98	1.04	1.17	<i>1.3</i> <i>base</i>	1.43	1.56	1.63	1.69
<i>Average log-sales</i>	12.94	12.98	13.02	<i>13.10</i>	13.19	13.26	13.32	13.36
<i>Average number of firms</i>	876	815	778	<i>713</i>	661	646	629	599
<i>Percentage of R&D firms</i>	0.16	0.21	0.25	<i>0.29</i>	0.37	0.46	0.51	0.57
<i>Average productivity</i>	6.19	6.21	6.22	<i>6.22</i>	6.24	6.25	6.26	6.27
<i>St. dev. of productivity</i>	0.47	0.47	0.47	<i>0.46</i>	0.46	0.46	0.45	0.44
<i>Average log-capital stock</i>	12.71	12.82	12.87	<i>12.92</i>	13.00	13.10	13.18	13.23
<i>St. dev. of log-capital</i>	0.68	0.64	0.62	<i>0.64</i>	0.65	0.67	0.70	0.70
<i>Capital-prod. correlation</i>	0.39	0.40	0.41	<i>0.41</i>	0.42	0.44	0.44	0.41
<i>Average investment rate</i>	0.10	0.12	0.12	<i>0.13</i>	0.13	0.14	0.14	0.14
<i>St. dev. of invest. rate</i>	0.22	0.22	0.21	<i>0.22</i>	0.23	0.26	0.28	0.27
<i>Entry rate (%)</i>	6.7	4.0	2.6	<i>1.7</i>	1.0	0.5	0.3	0.3
<i>Exit rate (%)</i>	6.5	3.8	2.3	<i>1.3</i>	0.5	0.1	0.1	0.0

Table XIV: Predicted moments from changes in entry costs.

are very small. The innovation costs play an important role when there is trade liberalization. Getting access to foreign markets allows the firms to exploit the economies of scale in R&D and introduce more innovations. Overall, sunk costs explain why R&D is done by larger firms and why successful firms need to invest in physical capital. The two results put together can explain the observed performance of some industries after trade liberalization episodes, like the Portuguese moulds industry.

There are some challenging questions left unanswered. The model proposed here is partial equilibrium. A trade liberalization event will certainly trigger other general equilibrium effects on wages and returns on capital. How these effects affect the innovation decisions is an interesting question to be considered. Second, the model has abstracted from import competition and strategic interactions. While this is a good approximation for the case studied here, it might be invalid in other scenarios.

A Appendix

A.1 Sample construction and descriptive statistics

The sample comes from three sources: Aggregate variables (revenues, value added, employment) come from the Portuguese National Statistics Office (INE); industry price deflators are collected from IAPMEI (2006); and the firm level sample was extracted from the Bank of Portugal database on firms across the economy (*Central de Balancos*, five digit NACE code industry 29563 - moulds industry).

Some notes on the *Central de Balancos*: The sample has been collected by the Central Bank since 1986. However, due to changes in accounting rules, it is only comparable from 1990. The data between 1990 and 1994 is not considered reliable. From 2000 the sampling method (simple random sampling) was changed to stratified sampling causing a drop in the number of observed (mostly smaller) firms in 1999 and 2000.

Representativeness: The sample is representative of the whole industry, in particular for the early periods. It covers 90% of total revenues and industry employment in 1994 and the coverage decreases to a minimum of 50% of revenues (40% of employment) in 2003. This reduction is mainly due to changes in the sampling procedure as explained above. There is an obvious gap in productivity trends between the sample and the industry (also total revenues and employment). Labor productivity increased by roughly 60% in the sample and only 40% in the industry. None of the aggregate variables are calculated using the sample but come directly from the collected industry wide variables.

Variable construction:

Capital stock was calculated using the perpetual inventory method with a 8% depreciation rate

$$K_{i,t+1} = (1 - depreciation) * K_{it} + I_{i,t+1}$$

Value added is equal to revenues subtracted from materials and external services expenditures

$$VA_{it} = Y_{it} - M_{it} - ESE_{it}$$

R&D dummy variable takes a value equal to one whenever positive R&D was reported in the past or present and zero otherwise.

Cash flow is constructed as the sum of accounting profits plus depreciation and amortization.

Both aggregate and individual revenues and value added were deflated with the industry price deflator.

In 11 observations the number of workers reported was zero and were dropped.

There were 9 holes identified in the sample, i.e. firms that interrupt reporting for 1 or more consecutive years. Either the earlier or later periods are dropped, minimizing the total number of observations lost.

	Mean	Std. Dev.	Min	Max
All firms: 1274 observations				
<i>Sales (EUR)</i>	1,574,073	2,869,201	3,292	34,700,000
<i>Exports (EUR)</i>	891,333	2,483,554	0	31,800,000
<i>Capital stock (EUR)</i>	1,058,104	2,130,734	135	23,800,000
<i>Employment</i>	32	39	1	258
<i>Labor productivity (EUR)</i>	20,381	9,044	359	74,632
<i>Investment rate</i>	0.20	0.25	0.00	5.32
<i>Sales growth</i>	8.9%	34.5%	-195.8%	469.0%
<i>Value added growth</i>	9.4%	40.5%	-289.3%	477.0%
<i>Labor productivity growth</i>	5.7%	37.1%	-289.3%	284.3%
Non R&D firms: 1009 observations				
<i>Sales (EUR)</i>	1,198,854	2,321,233	3,292	26,800,000
<i>Exports (EUR)</i>	640,879	1,919,257	0	25,200,000
<i>Capital stock (EUR)</i>	835,706	1,854,294	135	20,600,000
<i>Employment</i>	27	35	1	230
<i>Labor productivity (EUR)</i>	19,609	9,178	359	74,632
<i>Investment rate</i>	20.9%	27.5%	0.0%	531.7%
<i>Sales growth</i>	9.9%	37.9%	-195.8%	469.0%
<i>Value added growth</i>	10.4%	45.2%	-289.3%	477.0%
<i>Labor productivity growth</i>	6.2%	41.1%	-289.3%	284.3%
R&D firms: 265 observations				
<i>Sales (EUR)</i>	3,002,735	4,066,477	99,206	34,700,000
<i>Exports (EUR)</i>	1,844,947	3,811,178	0	31,800,000
<i>Capital stock (EUR)</i>	1,904,897	2,802,605	53,161	23,800,000
<i>Employment</i>	52	45	3	258
<i>Labor productivity (EUR)</i>	23,321	7,861	7,148	59,923
<i>Investment rate</i>	16.7%	14.2%	0.0%	77.5%
<i>Sales growth</i>	5.6%	20.1%	-101.8%	123.3%
<i>Value added growth</i>	6.3%	19.6%	-113.3%	102.4%
<i>Labor productivity growth</i>	3.9%	19.9%	-87.2%	116.9%
<i>R&D to sales ratio</i>	0.9%	3.4%	0.0%	46.5%

Source: Central de Balanços, Bank of Portugal

Table A.I: Summary statistics.

There are few observations on entry and exit. Due to the way the data was collected some firms might not be reported in the sample but still be active in the industry complicating the identification of entry and exit. Firms could have been operating in the market before first appearing in the sample and they might still be operating after leaving the sample. The problem is addressed with two variables to identify entry and exit. For entry, firms report their founding year and I consider it to be an entry if the firm first appeared in the sample within a 2 year window from the reported founding year (values reported in Table A.II under the column "entry"). Regarding exit the central bank collects a variable for the "status" of the firm. The quality of the exit variable is poor and some firms might have closed down and still be reported as "active", so only a fraction of actual exits is captured. I was able to identify a total of 48 entries and 7 exits from the panel.

<i>Year</i>	<i>Number of firms</i>	<i>Number of non R&D firms</i>	<i>Number of R&D firms</i>	<i>R&D start-ups</i>	<i>Entry</i>	<i>Entry in the dataset</i>	<i>Exits</i>
1994	144	134	10	-	2	3	0
1995	157	137	20	10	12	14	2
1996	165	141	24	4	8	14	0
1997	170	145	25	2	11	20	2
1998	164	135	29	7	9	33	0
1999	136	108	28	3	2	46	1
2000	92	68	24	7	2	8	0
2001	88	56	32	9	1	5	0
2002	88	53	35	4	1	2	0
2003	86	48	38	3	0	0	2
Total	1290	1025	265	49	48	145	7

Table A.II: R&D, Entry and Exit: number of observations per year.

A.1.1 Aggregate State

Market definition: The market is defined as total worldwide demand for Portuguese moulds. The total worldwide demand would be a better definition but observations for total world revenues are not available. The sample contains observations for total industry revenues (national plus exports). One restriction added by this market definition is that Portuguese firms take as exogenous the evolution of foreign competition. Let Y^W be the world supply of moulds, Y^{NP} the world supply of foreign moulds and $Y = Y^W - Y^{NP}$ the Portuguese supply of moulds. The definition restricts foreign competition to be exogenous and Y^{NP} to evolve exogenously.

Aggregate state definition: The aggregate state is defined as average deflated total industry revenues: $\frac{Y/N}{\tilde{P}}$. It can be divided into three components. The first is total industry revenues (Y) and can easily be assumed to evolve exogenously. As explained above, it is total demand for Portuguese moulds. The second variable is the industry price (\tilde{P}) and being the solution to the static pricing game it evolves endogenously. The pricing strategies are a mapping from states onto the pricing space. Finally, the total number of incumbents (N) is also endogenous and it depends on market size. Modeling the three variables separately would involve taking into account (and estimating) all cross correlations.

In Figure A.1 I plot the evolution of all variables. The market was growing between 1993 and 2000 and stopped until 2003. The number of firms share a similar pattern. On the other side prices were increasing slightly and decreased in the later years. The industry grew substantially after 1994 due to the strong increase in demand for Portuguese moulds. We also observe an increase in labor productivity and R&D. The cross correlations are as expected with prices being negatively correlated with number of firms and market size, and the number of firms being positively correlated with market size. The evolution of the three variables is summarized by the evolution of the single index variable, $\frac{\tilde{Y}}{\tilde{P}}$ (average deflated revenues).

	Number of firms	Production (EUR mio)	Exports (EUR mio)	Exports % of sales	Employment	Value Added (EUR mio)	Price (EUR/ton)
1994	644	171	132	77%	5,133	101	24.43
1995	570	193	151	78%	5,796	114	25.25
1996	452	244	191	78%	7,316	143	25.71
1997	477	293	220	75%	7,821	166	25.73
1998	461	322	232	72%	7,740	167	24.62
1999	549	362	250	69%	8,429	208	25.23
2000	604	412	277	67%	8,879	228	26.49
2001	612	421	328	78%	8,919	240	26.74
2002	722	378	310	82%	9,312	235	24.97
2003	738	403	303	75%	8,766	227	22.86
2004	1109	455	340	.	9,846	259	20.33
2005	1230	468	298	.	10,108	256	18.69

Source: National statistics office, INE (2007)

Table A.III: Aggregate variables

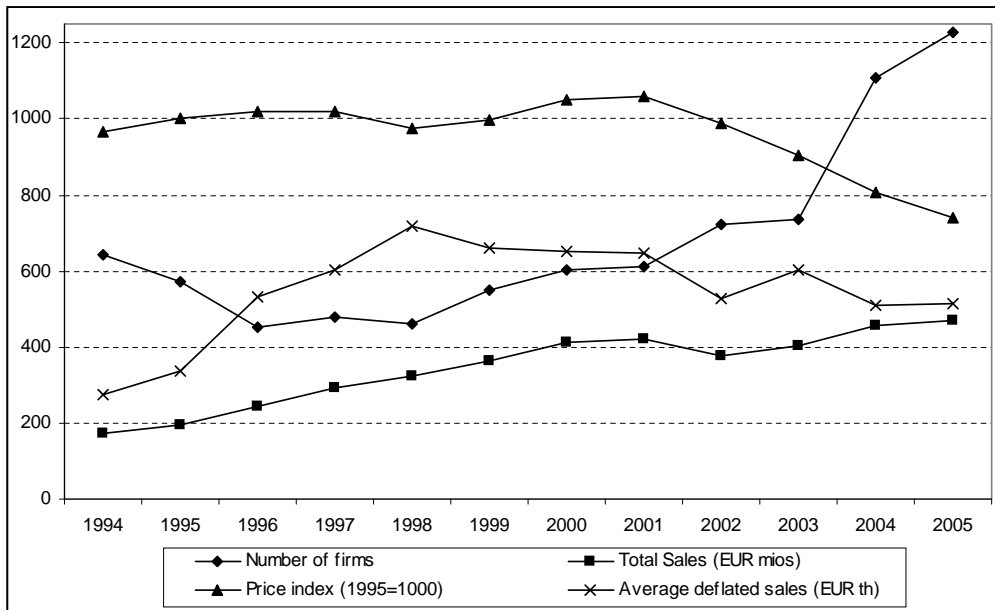


Figure A.1: Aggregate variables: Number of firms, price index, average and total sales

A.2 Production function estimation

Here, I explain how total factor productivity is estimated. Productivity is not directly observed and there are several available methods to estimate it as the residual from a production function. De Loecker (2007) proposes an estimator for both the production function parameters and the demand elasticity under imperfect competition when one uses deflated revenues instead of quantities (see also Klette and Griliches (1995)). The estimator, however, cannot be directly applied to an equilibrium model.¹⁶ Input demand functions depend on the industry state, more precisely, the aggregate industry state. For example investment functions depend on equilibrium beliefs about industry evolution. Specifying the investment policy as originally proposed by Olley and Pakes (1996), $i(\omega)$, is a misspecification since the equilibrium policy for investment in a Markov Perfect Equilibrium depends on the state of all players in the industry $i^*(\omega_1, \dots, \omega_N)$. Therefore, elasticity of demand cannot be recovered in the first step as proposed by De Loecker

¹⁶This is the true for most methods with similar approaches to Olley and Pakes (1996). Such methods look at single agent problems and disregard the possibility that policy functions (investment or materials) are equilibrium solutions to dynamic games. Therefore, the policy functions will be misspecified and the productivity control function will not include some important industry level variables leading to bias in the parameter estimates due to unobserved heterogeneity. How large this bias is in practice is an empirical question.

(2007) since the input demand is also a function of aggregate revenues.¹⁷ Using a Cobb-Douglas production function with labor and capital ($Q = e^{\omega_i} L^{\alpha_l} K^{\alpha_k}$), adding a demand function ($Q = \frac{\tilde{Y}}{\tilde{P}} \left(\frac{P}{\tilde{P}}\right)^{-\eta}$) with market size (\tilde{Y}) and price index (\tilde{P}) and taking the logs of firm level deflated revenues ($\frac{P_{it}Q_{it}}{\tilde{P}}$)

$$y_{it} - \tilde{p}_t = q_{it} + p_{it} - \tilde{p}_t = \frac{1}{\eta}(\tilde{y}_t - \tilde{p}_t) + \frac{\eta - 1}{\eta}(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}) \quad (\text{A.1})$$

Using materials to control for the unobservable as in Levinsohn and Petrin (2003), the input demand is a function of the state at time t (individual and industry states)

$$m_{it} = m(\omega_{it}, k_{it}, R_{it}, S_t) \quad (\text{A.2})$$

Assuming invertibility¹⁸

$$\omega_{it} = \omega(k_{it}, R_{it}, S_t, m_{it}) \quad (\text{A.3})$$

Since productivity is also a function of market conditions ($S_t = \frac{\tilde{Y}_t}{\tilde{P}_t}$) in equation (A.2), demand elasticity (η) is not identified in the first stage, because it enters nonlinearly in the control function (A.3). This is the main difference from a single agent framework as in De Loecker (2007) where input demand depends solely on individual state variables ($m_{it} = m(\omega_{it}, k_{it}, R_{it})$). Using the controlled first order Markov process assumption for productivity $\omega_{it} = E[\omega_{it} | \omega_{it-1}, R_{it-1}] + \nu_{it}^\omega$.

A.2.1 Stage I

From eq. (A.1) using deflated revenues as variables $y_{it}^p = y_{it} - \tilde{p}_t$ and $\tilde{y}_t^p = \tilde{y}_t - \tilde{p}_t = \ln(S_t)$ the production function is

$$y_{it}^p = \frac{\eta - 1}{\eta} \alpha_l l_{it} + \phi(k_{it}, R_{it}, \tilde{y}_t^p, m_{it}) + y_{it}^\varepsilon \quad (\text{A.4})$$

¹⁷There is also the selection problem due to exit as explained in Olley and Pakes (1996). In this paper I will abstract from this problem since the number of exits observed in the data is very small and the correction does not affect my estimates. However, if there were more observations on exits, this problem could be more carefully addressed.

¹⁸With imperfect competition an increase in productivity might not lead to an increase in output and therefore in materials usage causing concerns over invertibility. For the demand system specified, an increase in productivity is equivalent to a decrease in costs and it translates in a decrease in prices. Both total output and materials usage increase.

where y_{it}^ε is measurement error in y_{it}^p , and $\widehat{\frac{\eta-1}{\eta}\alpha_l}$ and $\hat{\phi}$ can be estimated nonparametrically or using an n th-order polynomial approximation from

$$\phi(k_{it}, R_{it}, \tilde{y}_t^p, m_{it}) = \frac{1}{\eta} \tilde{y}_t^p + \frac{\eta-1}{\eta} \alpha_k k_{it} + \frac{\eta-1}{\eta} \omega(k_{it}, R_{it}, \tilde{y}_t^p, m_{it})$$

A.2.2 Stage II

Let the estimated value be $\hat{\phi}_{it} = \hat{y}_{it}^p - \widehat{\frac{\eta-1}{\eta}\alpha_l} l_{it}$. An estimate of $\frac{\eta-1}{\eta} \omega_{it}$ for a given $\widehat{\frac{\eta-1}{\eta}\alpha_k}$ and $\frac{1}{\eta}$ is

$$\frac{\widehat{\frac{\eta-1}{\eta}\omega_{it}}}{\eta} = \hat{\phi}_{it} - \frac{1}{\eta} \tilde{y}_t^p - \frac{\widehat{\frac{\eta-1}{\eta}\alpha_k}}{\eta} k_{it}$$

Approximate $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$ with an n th-order polynomial $P^{n,\omega}(\omega_{it-1}, R_{it-1})$. Several approximations are used in the empirical section and in practice a cubic polynomial provides a good fit, in particular at the tails of the observations¹⁹

$$\hat{y}_{it}^p - \frac{\widehat{\frac{\eta-1}{\eta}\alpha_l} l_{it}}{\eta} = \frac{1}{\eta} \tilde{y}_t^p + \frac{\eta-1}{\eta} \alpha_k k_{it} + P^{n,\omega}(\omega_{it-1}, R_{it-1}) + \nu_{it}^\omega \quad (\text{A.5})$$

Finally, $\frac{1}{\eta}$ and α_k are estimated by nonlinear least squares using eq. (A.5).

A.2.3 Results

Estimates are reported in Table A.IV.²⁰ The estimated labor and capital coefficients are 0.51 and 0.4, while the estimated demand elasticity implies a price-cost margin of 8%. The values are at a reasonable level and within the range of parameters found in the literature for other industries. In columns (iii) and (iv) results are also reported for a linear and a sigmoid parametrization for productivity transition. The advantage is that both preserve monotonicity. Overall the differences are negligible and there is no evidence that functional forms are restrictive.

¹⁹Instead of using a cubic polynomial, also reported are the results with a sigmoidal function which preserves monotonicity: $E(\omega_{it}|\omega_{i,t-1}) = \frac{\gamma_0}{(1+\gamma_1 \exp(-\omega_{i,t-1}))}$

²⁰The results are reported for value added production functions. The approach is identical to the one using sales under the assumption that materials are a constant share of total sales. Since by definition $Y_{it} = VA_{it} + M_{it}$, if $M_{it} = \alpha_m Y_{it}$ then $Y_{it} = VA_{it} + \alpha_m Y_{it}$ so that $Y_{it} = \frac{1}{(1-\alpha_m)} VA_{it}$.

	(i)		(ii)		(iii)		(iv)		(v)		(vi)	
Equation:	1st stage		2nd stage		2nd stage		2nd stage		Fixed Effects		Fixed Effects	
Model:	(A.4)		(A.5)		(A.5)		(A.5)		(A.1)		(A.1)	
	OLS		NLLS		NLLS		NLLS		FE		FE	
			<i>Cubic</i>		<i>Linear</i>		<i>Sigmoidal</i>				<i>Control</i>	
			<i>Approx.</i>		<i>Approx.</i>		<i>Approx.</i>				<i>function</i>	
	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>	Coef.	<i>s.e.</i>
l_{it}	0.47	0.04							0.69	0.04	0.52	0.04
k_{it}			0.37	0.02	0.41	0.03	0.38	0.02	0.37	0.03		
\tilde{y}_t^p			0.08	0.04	0.07	0.05	0.07	0.04				
const.							3.57	0.64				
\hat{R}^2	98%		92%		92%		92%		92%		93%	
Obs.	1,271		1,044		1,044		1,044		1,271		1271	
Firms	227		223		223		223		227		227	
Labor coef.			0.51		0.51		0.74		0.69			
Capital coef.			0.40		0.44		0.40		0.37		-	
Price Cost Margin			8%		7%		7%		-		-	

Notes: Results for the first stage of the production function estimates are reported in column (i). Columns (ii)-(iv) report the second stage estimates using a cubic, linear and sigmoidal approximation, respectively. Column (v) reports the results for the fixed effects specification with year dummies. The specification in column (vi) is identical to column (i) with added fixed effects. Productivity transition parameters in columns (ii) to (iv) not reported.

Table A.IV: Production function estimates.

Results using alternative methods are also reported. In particular using a simple fixed effects specification with time dummies (column (v)) does not perform well due to serial correlation in productivity. Adding the control function $E[\omega_{it}|\omega_{it-1}, R_{it-1}]$ in column (vi) reduces the magnitude of the labor coefficient suggesting the fixed effects and time dummies are not properly capturing the correlation between labor and productivity

A.3 Figures

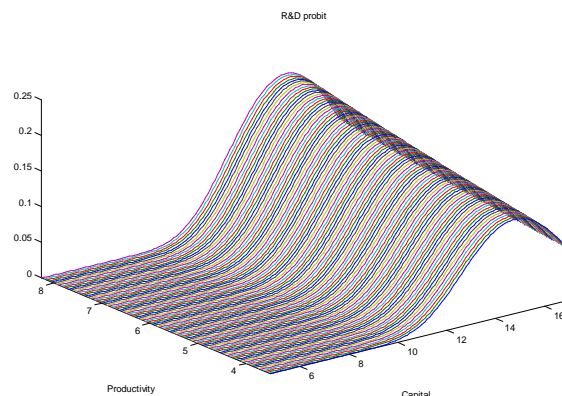


Figure A.2: R&D probit, fit in productivity and capital space

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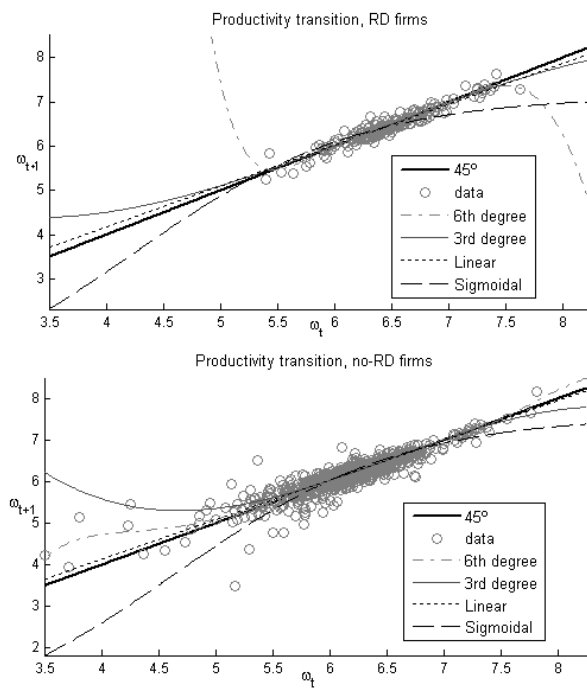


Figure A.3: Productivity transition (observations and fit)

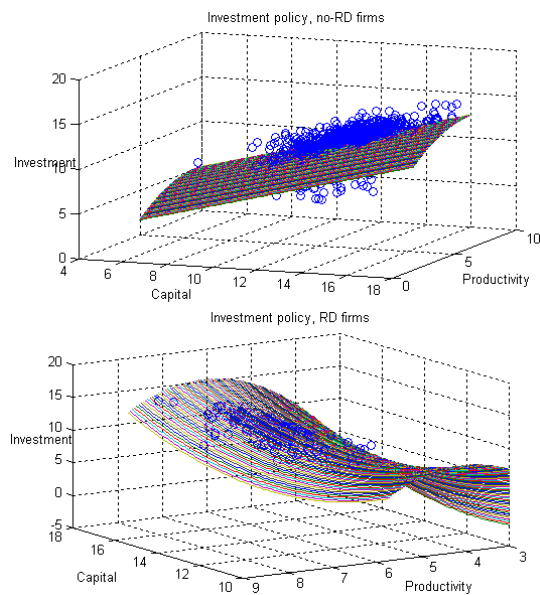


Figure A.4: Investment functions, fit in productivity and capital space

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B Technical Appendix (not for publication)

B.1 The aggregate state dynamic model

This section describes the elements of the general aggregate state dynamic model and characterizes the equilibrium conditions. I draw on Pesendorfer and Schmidt-Dengler (2008) and extend some results for the aggregate state model.

B.1.1 States and actions

Time is discrete, $t = 1, 2, \dots, \infty$. There are N players where a player is denoted by $i \in \{1, \dots, N\}$.

States Agents are endowed with a state variable $s_{it} \in \mathbf{S}_i = \{1, \dots, L\}$ and a vector of payoff shocks $\varepsilon_{it} \in \mathbb{R}^K$. Both the state and the shocks are privately observed by the players. The econometrician observes the states, s_{it} , but not the payoff shocks, ε_{it} .

The industry state is $\mathbf{s}_t = (s_{1t}, \dots, s_{Nt}) \in \mathbf{S} = S_i^N$. The cardinality of the state space is $m_s = L^N$. The state of competitors $s_{-it} = (s_{1t}, \dots, s_{i-1,t}, s_{i+1,t}, \dots, s_{Nt}) \in \mathbf{S}_{-i} = S_i^{N-1}$ with cardinality $m_{s-1} = L^{N-1}$. The vector of payoff shocks is drawn independently from the strict monotone and continuous distribution $F(\cdot | \mathbf{s}_t)$.

Actions Each player chooses an action $a_{it} \in A_i = \{0, \dots, K\}$. Decisions are simultaneous after players observe their state and payoff shock. A profile $\mathbf{a}_t = (a_{1t}, \dots, a_{Nt}) \in \mathbf{A} = A_i^N$ and $\mathbf{a}_{-it} \in \mathbf{A}_{-i} = A_i^{N-1}$. The cardinality of the action space \mathbf{A} is $m_a = (1 + K)^N$ and \mathbf{A}_{-i} is $m_{a-1} = (1 + K)^{N-1}$.

State transition The individual state transition is described by a density function $p : A_i \times S_i \times S_i \rightarrow [0, 1]$. A typical element of $p(a, s, s')$ equals the probability that state s' is reached from state s when action a is chosen and $\sum_{s' \in S_i} p(a, s, s') = 1$. The industry transition can be represented by a density function $g : \mathbf{A} \times \mathbf{S} \times \mathbf{S} \rightarrow [0, 1]$. A typical element of $g(\mathbf{a}, \mathbf{s}, \mathbf{s}') = \prod_{j=1}^N p(a_j, s_j, s'_j)$ equals the probability that state \mathbf{s}' is reached from state \mathbf{s} when action profile \mathbf{a} is chosen and $\sum_{\mathbf{s}' \in \mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') = 1$.

Per period payoff Firms receive per period returns which depend on the state of the industry, current actions and shocks ($\pi(\mathbf{a}_t, \mathbf{s}_t, \boldsymbol{\varepsilon}_{it})$).

Assumption B.1 (a) *There exists a function ($S : \mathbf{S} \rightarrow J$) that maps the vector of firm's individual states (\mathbf{s}_t) into an aggregate index ($S(s_{1t}, \dots, s_{Nt}) \in J$). The cardinality of J is $m_J \leq m_s$.*

(b) *Per period payoffs can be written as*

$$\pi(\mathbf{a}_t, \mathbf{s}_t, \boldsymbol{\varepsilon}_{it}) = \pi(\mathbf{a}_t, s_{it}, S_t) + \sum_{k=1}^K \varepsilon_{it}^k \cdot 1(a_{it} = k)$$

Under this assumption, S_t is the payoff relevant variable commonly observed by all agents. Notice that the payoff relevant shocks ($\boldsymbol{\varepsilon}_{it}$) have no impact on the stage game pricing. One type of demand which meets this assumption is Monopolistic Competition.

Assumption B.2 (a) *(Incomplete Information) Individual states and actions are private information and;*

(b) *(Markov beliefs) players use the current state to form their beliefs: $\sigma(\mathbf{s}|S, s_i)$.*

Under Assumption B.2 the only common information is the aggregate state. Moreover, it restricts agents to form beliefs using only the current state. This is the main assumption that distinguishes the aggregate state model from the literature with no privately observed state variables. The restriction on beliefs is important to circumvent the learning problem that would emerge otherwise.

Strategies I consider pure symmetric Markovian strategies, $a_{it}(s_{it}, S_t, \boldsymbol{\varepsilon}_{it})$. These strategies depend only on payoff relevant variables. Using symmetry we can drop the i subscript and imposing stationarity we can drop the t subscript:

$$a_{it}(s_{it}, S_t, \boldsymbol{\varepsilon}_{it}) = a(s_{it}, S_t, \boldsymbol{\varepsilon}_{it})$$

Let $p(a_i|s_i, S; \sigma)$ denote the probability of player i choosing action a_i when he observes state (s_i, S) and has beliefs σ .

Value function We can write the ex-ante value function defined as the discounted sum of future payoffs before player specific shocks are observed and actions taken, as

$$V(s_i, S; \sigma) = \sum_{\mathbf{s} \in \mathbf{S}} \sum_{\mathbf{a} \in \mathbf{A}} p(\mathbf{a}|\mathbf{s}, S; \sigma) \sigma(\mathbf{s}|s_i, S) \left[\pi_i(\mathbf{a}, \mathbf{s}) + \beta \sum_{s'_i, S'} \tilde{g}(\mathbf{a}, \mathbf{s}, s'_i, S') V(s'_i, S'; \sigma) \right] \quad (\text{B.1})$$

$$+ \sum_{k=1}^K E_\varepsilon[\varepsilon_i^k | a_i = k] \cdot p(a_i = k | s_i, S; \sigma)$$

where $\tilde{g}(\mathbf{a}, \mathbf{s}, s'_i, S') = (\sum_{\mathbf{s}' \in \mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') \cdot \mathbf{1}(S' = S(\mathbf{s}'), s'_i \in \mathbf{s}'))$, $p(\mathbf{a}|\mathbf{s}, S; \sigma) = \prod_{j=1}^N p(a_j | s_j, S; \sigma) \cdot \mathbf{1}(S = S(\mathbf{s}), s_j \in \mathbf{s})$ and E_ε is the expectation over the payoff shocks. Equation (B.1) is satisfied at each $(s_i, S \in \mathbf{S}_i \times J)$. This can be written in matrix form.

$$\begin{aligned} \mathbf{V}(\sigma) &= (\sigma \cdot \mathbf{P}(\sigma)) \mathbf{\Pi} + \mathbf{D}(p(\sigma)) + \beta (\sigma \cdot \mathbf{P}(\sigma)) \tilde{G} \mathbf{V}(\sigma) \\ &= [\mathbf{I}_{LJ} - \beta (\sigma \cdot \mathbf{P}(\sigma)) \tilde{G}]^{-1} [(\sigma \cdot \mathbf{P}(\sigma)) \mathbf{\Pi} + \mathbf{D}(p(\sigma))] \end{aligned} \quad (\text{B.2})$$

where $\mathbf{V}(\sigma) = [V(s_i, S; \sigma)]_{s_i, S \in \mathbf{S}_i \times J}$ is the $(L \cdot m_J) \times 1$ dimensional vector of expected discounted sum of future payoffs. \tilde{G} is the $(m_a \cdot m_s) \times (L \cdot m_j)$ transition matrix with element $\tilde{g}(\mathbf{a}, \mathbf{s}, s'_i, S')$. $\mathbf{\Pi}$ is the $m_a \cdot m_s \times 1$ dimensional vector of profits, $\mathbf{P}(\sigma)$ is the $(L \cdot m_J) \times (m_a \cdot m_s)$ dimensional matrix with element in row (s_i, S) and column (\mathbf{a}, \mathbf{s}) equal to $p(\mathbf{a}|\mathbf{s}; \sigma) \cdot \mathbf{1}(s_i \in \mathbf{s}, S = S(\mathbf{s}))$. σ is the $(L \cdot m_J) \times (m_a \cdot m_s)$ dimensional matrix with element in row (s_i, S) and column (\mathbf{a}, \mathbf{s}) equal to $\sigma(\mathbf{s}|s_i, S) \cdot \mathbf{1}(S = S(\mathbf{s}), (s_i) \in \mathbf{s})$ for any \mathbf{a} . $(\sigma \cdot \mathbf{P}(\sigma))$ is the element by element product of the two matrices. I_J is the $L \cdot m_J$ dimensional identity matrix. Finally, $D(p(\sigma))$ is the $(L \cdot m_J) \times 1$ dimensional vector of expected payoff shocks.

B.1.2 Equilibrium

Let $u^S(a_i; \sigma)$ denote the continuation value net of payoff shocks under action a_i and beliefs σ .

$$u^S(a_i; \sigma, \theta) = \pi(a_i, s_i, S)$$

$$+\beta \sum_{\mathbf{s}_{-i} \in \mathbf{S}_{-i}} \sum_{\mathbf{a}_{-i} \in \mathbf{A}_{-i}} p(\mathbf{a}_{-i} | \mathbf{s}, S; \sigma) \sigma(\mathbf{s} | s_i, S) \sum_{\mathbf{s}' \in \mathbf{S}} g(\mathbf{a}, \mathbf{s}, \mathbf{s}') \cdot \mathbf{1}(S' = S(\mathbf{s}')) V(s'_i, S'; \sigma) \quad (\text{B.3})$$

where $p(\mathbf{a}_{-i} | s_i, \mathbf{s}_{-i}, S; \sigma) = \prod_{j=1, j \neq i}^N p(a_j | s_j, S; \sigma) \cdot \mathbf{1}(S = S(s_i, \mathbf{s}_{-i}), s_j \in \mathbf{s}_{-i})$. The optimality condition requires that action a_i is chosen when

$$u(a_i; \sigma, \theta) + \varepsilon_i^{a_i} \geq u(a'_i; \sigma, \theta) + \varepsilon_i^{a'_i} \quad \text{for all } a'_i \in \mathbf{A}_i \quad (\text{B.4})$$

Writing equation (B.4) as the ex-ante optimal choice probability

$$\begin{aligned} p(a_i | s_i, S; \sigma) &= \Psi(a_i, s_i, S; \sigma) \\ &= \int \left[\prod_{k \in \mathbf{A}_i, k \neq a_i} \mathbf{1}(u(a_i; \sigma) - u(k; \sigma) \geq \varepsilon_i^k - \varepsilon_i^{a_i}) \right] dF \end{aligned} \quad (\text{B.5})$$

We can write this system of $(K \cdot \mathbf{S}_i \cdot J)$ equations in vector form

$$\mathbf{p}^\sigma = \Psi(\mathbf{p}^\sigma)$$

A solution exists by Brouwer's fixed point theorem since this equation is a continuous self-map on $[0, 1]^{K \cdot \mathbf{S}_i \cdot J}$ as shown in Schmidt-Dengler and Pesendorfer (2008). The solution is the optimal behavior for given beliefs, \mathbf{p}^σ .

Stationary distribution Let $\lambda(\mathbf{s}; \sigma)$ be the probability of being in state \mathbf{s} and let the transition be

$$\lambda(\mathbf{s}'; \sigma) = \sum_{\mathbf{s}} \sum_{\mathbf{a}} p(\mathbf{a} | \mathbf{s}; \sigma) g(\mathbf{a}, \mathbf{s}, \mathbf{s}') \lambda(\mathbf{s}; \sigma) \quad (\text{B.6})$$

Define \mathbf{P}^σ as the $m_s \times (m_s \cdot m_a)$ dimensional matrix consisting of $p(\mathbf{a} | \mathbf{s}; \sigma)$ in row \mathbf{s} , columns (\mathbf{a}, \mathbf{s}) and zero in the remaining columns. G is the $(m_a \cdot m_s) \times m_s$ dimensional matrix. Convergence to a unique stationary distribution with $\lambda^*(\mathbf{s}; \sigma) > 0$ for all $\mathbf{s} \in \mathbf{S}$ occurs if $\mathbf{P}^\sigma G$ is a regular matrix. We can write equation (B.6) in vector form as

$$\lambda^{\sigma'} = \mathbf{P}^\sigma G \lambda^\sigma$$

and the stationary distribution is the eigenvector $(\lambda_{\mathbf{s}}^\sigma)$, solving

$$(\lambda_{\mathbf{s}}^\sigma)^* \cdot (I_{m_s} - \mathbf{P}^\sigma G) = 0 \quad (\text{B.7})$$

The stationary conditional distribution is

$$\begin{aligned} \lambda^*(\mathbf{s}|S; \sigma) &= \frac{\Pr(\mathbf{s}, S|\sigma)}{\Pr(S|\sigma)} = \frac{\Pr(S|\mathbf{s}; \sigma) \Pr(\mathbf{s}|\sigma)}{\Pr(S|\sigma)} \\ &= \frac{\mathbf{1}(S = S(\mathbf{s})) \cdot \lambda^*(\mathbf{s}; \sigma)}{\sum_{\mathbf{s} \in \mathbf{S}} [\mathbf{1}(S = S(\mathbf{s})) \cdot \lambda^*(\mathbf{s}; \sigma)]} \end{aligned} \quad (\text{B.8})$$

$$\lambda^*(\mathbf{s}|s_i, S; \sigma) = \frac{\Pr(\mathbf{s}, s_i, S|\sigma)}{\Pr(s_i, S|\sigma)} = \frac{\mathbf{1}(S = S(\mathbf{s})) \mathbf{1}(s_i \in \mathbf{s}) \lambda^*(\mathbf{s}; \sigma)}{\sum_{\mathbf{s} \in \mathbf{S}} [\mathbf{1}(S = S(\mathbf{s}), s_i \in \mathbf{s}) \cdot \lambda^*(\mathbf{s}; \sigma)]}$$

and $\lambda^*(\mathbf{s}; \sigma)$ is probability of element $\mathbf{s} \in \mathbf{S}$ occurring under the stationary distribution $(\lambda_{\mathbf{s}}^\sigma)^*$. We can write this in matrix form

$$\boldsymbol{\lambda}_{\mathbf{s}|s_i, S}^* = \boldsymbol{\Lambda}(\boldsymbol{\sigma})$$

where $\boldsymbol{\lambda}_{\mathbf{s}|s_i, S}^*$ denotes a $(m_s \cdot S_i \cdot J) \times 1$ dimensional vector. In equilibrium, beliefs must be consistent with optimal behavior $\boldsymbol{\lambda}_{\mathbf{s}|s_i, S}^* = \boldsymbol{\sigma}$

$$\boldsymbol{\lambda}_{\mathbf{s}|s_i, S}^* = \boldsymbol{\Lambda}(\boldsymbol{\lambda}_{\mathbf{s}|s_i, S}^*) \quad (\text{B.9})$$

Equation (B.9) is both a necessary and sufficient condition for a Markov perfect equilibrium.

Theorem 1 *An equilibrium exists.*

Equation (B.9) gives a continuous self-map on $[0, 1]^{m_s \cdot S_i \cdot J}$. Brouwer's fixed point theorem implies there exists (at least) a fixed point $\boldsymbol{\lambda}^*$ to the function $\boldsymbol{\Lambda}$. This fixed point corresponds to an equilibrium.